Hands-On Math
Version 3.0 TZ

Mathematics Teaching Resource Manual

Tanzania

May 6, 2018

Questions or Comments?

Thank you for using the *Hands-On Math* manual! If you have any questions, comments, or would like to request a copy of this manual, please use the contact information given below.

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About This Book

Hands-On Math is a teaching resource manual for U.S. Peace Corps Volunteers serving in Tanzania. Many of these Volunteers work in schools having insufficient math teachers and teaching resources and all are new to the challenges of teaching in Tanzanian secondary schools. This manual was developed as a guide to assist Volunteers in teaching mathematics in Tanzania more effectively.

Like science, math must be learned by discovery. Mathematics is central to all sciences, and as such requires tangible interaction in order to achieve a full understanding of its concepts. For too long have ancient methods of lecture and rote memorization failed to provide students with a meaningful grasp on the principles of mathematics. In order to inspire students to take ownership of their education, teachers must use any and all available resources to convey the importance, joy and usefulness of learning math. But these resources need to be neither expensive nor difficult to obtain. The vast majority of the teaching aids and activities provided in this book can be utilized at little or no cost and are readily available even in the most rural village settings. This means that students themselves can and should be encouraged to use as many of them as possible so that they can see how the entire world is a forum for math learning.

In addition to fulfilling its own its own academic endeavors, the study of mathematics develops within its students a logical and systematic approach to problem-solving that may be applied to all facets of life. It is in this sense that math infiltrates all fathomable aspects of our lives and affirms itself as a fundamental component in the ongoing process of education. But logic is not necessarily inherent to an individual's mental ability. Instead it must be taught through puzzles, games, stimulating questions and problems applicable to real life. As students gain familiarity taking on certain kinds of problems and identifying suitable approaches to solving them, they increase their capacity for learning and become better adept at independently facing challenges they may come across in the future. Thus, many of the activities presented in this book are not only fruitful in teaching math, but also in developing thinking individuals.

Because many of the challenges faced by math teachers in Tanzania can differ drastically from those in an American context, this book offers some possible strategies for meeting said challenges from a Volunteer's perspective. However, there is no formula for teaching, regardless of location, and so these suggestions should be taken as nothing more than a guide based on the experiences of others. The value of a teacher lies in her ability to address the specific needs of her students, and so any teaching methodology should be open to adaptations and amendments upon further discovery of one's school and environment.

In addition to gaining a familiarity with methods of teaching math in a Tanzanian context, Volunteers must be aware of current exam regulations provided by the Ministry of Education and Vocational Training (MoEVT) in order to properly assist and prepare their students for the NECTA national examinations. To aid the Volunteer, a section has been added to this book outlining the most recently released exam guidelines released by the MoEVT. Additionally, a thorough collection of past NECTA exam problems spanning from 1995 to 2012 has been included and organized according to the current mathematics syllabus of topics from Form I up to Form IV. This is intended to assist the Volunteer in providing relevant examples of problems that students will be accountable to answer on their national examinations.

This book was originally created and published under the enthusiastic inspiration of PCV Marilyn Bick, with meaningful contributions coming from PCVs Cindy del Rosario, Julia Meyers, Teresa Ring and Katherine. Much of the inspiration and labor in converting the original manual into LaTeXwas provided by PCV Dave Berg. Now in its second edition, it has been reorganized in order to better address the needs of Volunteers. Additional content has been added based on inspiring and successful teaching techniques employed by PCVs Jon Mortelette, Sara Tomaskiewicz, John Clay and Belle Archaphorn, among many others. While advancing through their service in Tanzania, Volunteers will likely come across additional resources to contribute in order to further the scope of this book. This is highly encouraged and necessary for its continued relevance in the context of Peace Corps Volunteers teaching math in Tanzania. Volunteers wishing to make contributions, criticisms, concerns or suggestions to this manual

can do so using the official Volunteer website of Peace Corps Tanzania (pctanzania.org) or by following the contact information provided below.

In the words of Marilyn Bick,

TEACHING MATHEMATICS IS FUN
ENJOY TEACHING
INVOLVE THE STUDENTS
BE CRAZY SOME DAYS
LEARNING WILL HAPPEN
NUMBERS ARE TO BE PLAYED WITH DAILY!

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Contents

1 About This Book **3**

2 General Advice for Math Teachers **6**
2.1 Assessment . . . 6
2.2 Time, Time, Time . . . 6
2.3 Board Work . . . 6
2.4 Group Work . . . 7
2.5 Homework . . . 7
2.6 Critical Thinking . . . 7

3 Challenges Facing Math Teachers and Strategies for Success **9**
3.1 The Actual Teaching Part . . . 9
3.2 Creating a Positive Math Culture . . . 14

4 Form I-IV Topics and Activities **18**
4.1 Mathematics Activities for Form I . . . 18
4.2 Mathematics Activities for Form II . . . 25
4.3 Mathematics Activities for Form III . . . 27
4.4 Mathematics Activities for Form IV . . . 31

5 Games and Puzzles for Stimulating Interest in Mathematics **35**
5.1 Games . . . 35
5.2 Stimulating Puzzles . . . 39
5.3 Brain Teasers and Mind Benders . . . 41

6 Hands-On Tools from Locally Available Materials **43**
6.1 Accounts . . . 43
6.2 Cartesian Plane . . . 43
6.3 Circles . . . 44
6.4 Clinometer . . . 44
6.5 Compass . . . 45
6.6 Dice . . . 45
6.7 Dominoes . . . 45
6.8 Flash Cards . . . 45
6.9 Fractions and Decimals . . . 46
6.10 Geoboard . . . 46
6.11 Globe . . . 47
6.12 Number Line . . . 47
6.13 Protractor . . . 47
6.14 Spinners . . . 48
6.15 Three-Dimensional Objects . . . 48

7 Glossary of Math Terms in Kiswahili **50**

General Advice for Math Teachers

Every school, site, and volunteer experience is different. Variables such as class/school size, quality and quantity of past math teachers, available resources, location (rural/urban) and school type (day/ boarding, government/private) can all influence a volunteer's teaching approach and experience, so take the tips in this book as a guide only, and not a recipe for teaching success. It is advisable to take plenty of time and lots of observation of your school, classrooms, and students to see what works best for you and your students.

2.1 Assessment

Upon arriving at your school, you may find it helpful to do an overall assessment of the "math situation" present. It's good to get a feel for the current status of your school's math department, as well as the attitudes towards mathematics from the students' and teachers' perspectives.

You may come to a school that has never had a math teacher, or hasn't had a math teacher in a few years, or has a math teacher that has not taught regularly. As a result, many of your students may be behind in the syllabus, and it is helpful for you to know what they have and have not learned (or have and have not retained).

Before you start teaching, it may be helpful to test all of your students to see where their math skills are. For example, test your Form I's for basic math skills they should have obtained in primary school. For Form II's, give them questions from their Form I math textbook, and so on. Make the test short and simple, as it will likely take them much more time to complete than you expect! The results will show, in the very least, what topics you will need to spend time reviewing.

If you are teaching Form III's and IV's, it may also be helpful to look at math scores on the Form II NECTA exam. These will be helpful both in identifying students with strong math skills as well as the weaker students, and will give you a baseline to monitor student improvement later on. This may also show if there are opportunities for you to tailor your classes to the students' pace and ability. For example, if your school has more than one stream, ask your headmaster if they can be separated based on math ability; this will allow you to move at a faster pace with students that are good at math and spend more time with those that pick up concepts more slowly. If students are very far behind and it is possible to redistribute them out of form for math lessons, that can also be effective (although this requires a lot more effort to rework the school timetable).

2.2 Time, Time, Time

You cannot expect Tanzanian students to work as quickly or efficiently as you would hope. Many of them do not have times tables well memorized or basic math in their heads. Unless you plan on drilling them to work on speed, give students plenty of time to write notes, answer questions, and solve problems. They need additional time to translate an answer in their heads from Kiswahili to English, so be patient when you ask a question in class (wait several seconds). Also be very conscious of length when writing an exam. Pre-testing your students is also helpful in gauging how long they take to solve problems before giving your first real exam.

2.3 Board Work

Students love to answer questions they know! You will probably find that they are excited to raise their hands and come solve a problem on the board, but only if they are confident and know the answer. So start with easier questions to build their confidence before going on to more challenging problems. Learn their names, who the bright and weak students are, and call on a variety of them to solve problems. This will also help them stay awake and focused, if they know that they may be called on to solve a problem

at any time. Be sure to offer plenty of praise for correctness and help a student work through a problem if they are struggling. You may wish to use your better students to walk through an example on the board in Kiswahili after you've explained it in English. This can be helpful in getting the students to understand a concept, but be cautious of the likely possibility that they will soon stop listening to you and just wait for the Kiswahili explanation! In addition, board races between students are really fun for the students and also a great way to reinforce a topic and increase their problem-solving speed.

2.4 Group Work

The importance of group work cannot be stressed enough! You cannot lecture all period (especially during double periods) and hope to keep their attention. Try to finish the lecture early and give the rest of the class period to work on exercises. Groups of 2-4 students are usually best, and you can count off students to divide them into random groups. It can take time and be chaotic in larger classes as students move to form groups, so try to make a silent game out of it: students race to find and form their group using only hand gestures and mouthing words.

Group work can be very helpful with the language barrier, as students can discuss and work through problems together in Kiswahili. It is truly exciting to observe a class where several groups are working together and solving problems. Your stronger students can help explain concepts to the slower students, and if one group finishes early, they can go help other groups. You are also there to walk around and help students and check their answers. Many groups will have the same errors, so this is a great time to gauge what topics students understand well or struggle with, and you can help clarify concepts on the board.

2.5 Homework

Your decision of whether or not to assign homework will likely depend on the time available after class for you and your students. If you are teaching at a boarding school, students may have ample time to complete homework in the evenings, but students at day schools (especially girls) may have a lot of work around the house to attend to after school. In addition, your own teaching schedule may or may not permit you to dedicate the time necessary to grade all of the students' notebooks on a regular basis.

As an alternative to assigning homework, you may elect to give students time at the end of class to finish in-class exercises on their own and then go over the answers at the beginning of the next class. Often what you may find after marking an entire class's homework assignment is that the majority of the students either did not bother to complete the assignment or have simply copied answers from one another, which does not give you meaningful feedback as a teacher. Thus, you may find it more effective to be present when giving students exercises to work on, and to gauge student progress and understanding from group work or quizzes and tests.

Whatever you decide in terms of assigning homework, remember: math = practice! The only way for students to gain confidence and succeed in math is to get practice doing exercises *by themselves*! Group work is a great method for improving students' understanding, but they will not have anyone to hold their hands during the NECTA exam, and eventually must be comfortable with their own individual math abilities.

2.6 Critical Thinking

Mathematics is the best subject to teach the process of thinking since math is very logical. Logic is a step by step process to take given data and to produce the desired results. The step by step process needs to be taught to the students. If it is taught to students early on in mathematical studies, the students will experience success in mathematics. Math should not be memorized but processed through thinking. When thinking and processing data, math can be easy and fun.

Critical thinking is basic to real learning, as opposed to pure memorization. It begins early as students understand that 3 is greater than 2. The student should use a number line or objects to understand this relational concept. The student progresses in critical thinking to more complex problems. Thus, the student can more easily compare or contrast in other subjects if she or he learns to reason in mathematics. There are many math games and races to increase math thinking skills. The process does in fact need to be taught! Lessons need to be developed that teach students a step by step process of thinking.

Challenges Facing Math Teachers and Strategies for Success

3.1 The Actual Teaching Part

During a volunteer's service, he or she may encounter a variety of challenges specific to teaching mathematics in Tanzania. Of course, the strategies for meeting these challenges depends entirely on the individual, but given here are some possible approaches based on the experiences of previous volunteers.

3.1.1 Language Barriers

All Tanzanian secondary schools are supposed to be taught in an English medium. Nonetheless, almost all students are learning English as a second or third language, and therefore struggle to understand an English-speaking volunteer. This frustration can lead students to quickly give up, especially in a subject that they already deem to be difficult such as mathematics.

Strategies:

- *Keep at it!*: It may seem that, even after a long time, students are not catching on to English and don't understand a word of what you are saying. Repeated poor test scores may encourage you to give up on using English and resort to teaching in Kiswahili. Depending on the situation, this may be the best option, but before dropping English with your students, remember that learning is a very long process. The only way for them to improve their English skills is to hear and speak it everyday, and your class may be the only opportunity for them to practice. Simply showing your students that you are not willing to give up on them but rather continue to challenge them to learn English can be an inspiration and possibly motivate them to follow your example and not give up on themselves. Besides, you may find that whether you speak in English or Kiswahili, the students who want to learn will continue to make an effort, while those who are unmotivated will not become extremely studious just because you are speaking their language.
- *Show, don't tell*: Mathematics can be thought of as a universal language of its own. When it comes to teaching math (especially to Form I students), observation and practice can be much more beneficial tools for learning than lecturing. However, many students have developed a tendency to learn by rote memorization, so it is very important to use as many examples as possible in class to illustrate all of the distinctions in a new topic or principle.
- *Use student helpers*: Having well-performing students assist you in explaining a concept in Kiswahili can help others to understand more clearly, although if used too much, this strategy can lead students to stop listening to you altogether if they know a Kiswahili explanation is on the way.
- *Group work*: Getting students to work in groups gives them a forum to discuss problems and ask each other questions in their own language. As a teacher, you need to be present during this time to offer additional help and to make sure the students stay on task.

3.1.2 Slow learners / Unmotivated students

You will almost certainly find that your students have a wide range of math abilities, and that there are many who struggle to move at a pace you would like. This may be due to several factors: they never had a math teacher in primary school, they have problems at home interfering with their studies, they are just lazy, etc. Regardless of the reason, you will need to decide how to manage a class full of students moving at very different speeds.

Strategies:

- *Practice, Practice, Practice*: The content of mathematical concepts builds upon itself. First a child needs an awareness of numbers (e.g. the child has 2 brothers). Then a student learns operations

on numbers, which builds off of number awareness, and so on. The student uses what he or she knows to understand new mathematical concepts.

Therefore, it is so important to reinforce what is being taught. The student needs to practice all types of problems related to new knowledge learned. The teacher must drill, drill, drill: short quizzes or quick games are very helpful to ensure the progression of learning math skills.

As the math student progresses, he or she will be able to do more complex problems using many concepts taught in previous math classes. The student will be successful and enjoy math in school and in daily life.

- *REVIEW!*: Every class period should begin with a review from the previous class. This helps to reinforce every topic, allows students who were absent a chance to see what they missed, and provides a great opportunity to engage the students at the start of class with a board race or review game to get them excited for the day's math lesson.
- *Stream by ability*: If your school uses multiple streams, you may want to suggest streaming each form by student ability. That way, you can move more quickly with the fast learners and spend more time with the slow learners to accommodate each student's ability in math.
- *Focus on motivated students*: If you have a large number of students or classes to teach, you may find it in your best interest to focus your attention on the students who are very motivated to learn mathematics. You may find that many students are not willing to make the effort required to improve their own abilities. It's true that you are their teacher and may want to help every student, but at the same time you cannot force them to put forth an effort, and even if you could, doing so would not help them to take responsibility for their own education. You only have so much time to dedicate towards such a large number of students, so you must decide how to best utilize the time you have.

3.1.3 Ongoing assessment

It can be difficult to monitor student progress with such large class sizes and limited class time. However, receiving continuous feedback from your students is necessary to your success as a teacher.

Strategies:

- *Homework*: Homework may or may not be a feasible option at your school depending on what the students' after-school schedules are like. They may have a large number of domestic responsibilities after classes that eat up their time, but if not, homework assignments can be a great way to give students additional practice and can help you to identify how well the class has understood a concept. Marking hundreds of notebooks can be a daunting task for the teacher as well, so be sure to take your own time into account when assigning homework. If possible, try to give students at least 2 problems to work on by themselves after each new topic: one simple question to check for understanding, and one question to challenge them on something you didn't explicitly go over in class.
- *Pre- and post-tests*: For each chapter, give the students a pre-test to see if they already know a certain concept and you don't have to go over it in as much detail in class. Then at the end of the chapter give a post-test to see how much they improved from before your teaching. Show the results side by side to the students so they can see the progress they are making! A great motivation for the students and for you as a teacher!
- *Review games*: Games can be a wonderful way to engage students while getting useful feedback on what they have learned. Competitive games (especially boys vs. girls) will ensure that the students are trying their hardest to solve problems, so you can test them with questions of varying difficulty to see where they need improvement as a class.
- *In-class assessment*: The size of your classes may dictate whether or not this is feasible on a regular basis, but assigning problems and checking answers in class is helpful in that you have immediate feedback on a topic that is fresh in the students' minds. You also are able to watch the students to see if they are copying off one another, and you may get an idea for who the fast learners are by the students who consistently are the first to finish.

- *Pop quizzes*: This can be helpful if you are concerned that only a small number of your students are paying attention and doing their work. Students will be more likely to pay attention every day if they know there is the possibility of having a pop quiz on the topic at any time. Nothing like striking fear into the hearts of children to motivate math learning!

3.1.4 Class size / management

Tanzanian classes can typically run upwards of 60-80 students, if not more. Regardless of your teaching background, you will likely need to develop new strategies for managing such a large class in order to utilize your time effectively. Large class sizes can make it difficult to accomplish certain tasks like grading homework and in-class assignments, keeping track of attendance, and making sure every student is taking notes during class.

Strategies:

- *Group work*: There are many thinking styles of different students. Using group work helps to assist various patterns of learning, and so large classes can actually facilitate more learning if group work is used. The students need to be taught to work together as a team, though each student should write to facilitate individual learning. Math can not be discussion only, but the step by step thought process must be written by each student. The students should work as a team to accomplish their math calculations. When a group is finished, the answer is written on board and explained by a student. Questions are asked and the problem is discussed by the class.

  As the teacher moves around the large classroom, he or she is reinforcing good behavior. Also the teacher can mark or write a motivating comment in the exercise book. Student love this! If a group needs help, frequently a neighboring student can be asked to teach the group who needs help. This technique can be very helpful in a large classroom.

- *Utilize class monitors*: Each class should have a monitor and a monitress, who should be responsible for keeping track of attendance in class everyday. You can use these students to help you track who is or is not coming to class on a regular basis, and also to bring student notebooks to and from your office for grading assignments.

3.1.5 Teaching methodology

Every teacher has different preferred teaching methods. Yours will likely result from trial and error and seeing what works best for your students.

Strategies:

- *Discovery*: One of the best mathematics teaching techniques is to teach the students to discover formulas that are being learned by guided questions. For example, they can work many examples by picture to find perimeter of a rectangle. After adding length, width, length and width many times to find perimeter, the students can be guided by questions to discover $2l + 2w = P$. In this method of math teaching, the student will understand and remember the formula rather than memorizing. All formulas can be discovered because the formula does work, so it must have meaning. If the teacher is consistent in teaching through discovery, the teacher will be delighted with questions and discovery of his or her students. Learning is happening!! Teaching is successful!!

  Math learning is increased when senses are used. To use Math Tools, the students are using sight and touch. To use a cylinder 3-D object, the students can discover the formula for surface area. This area is found by taking the area of a rectangle around the cylinder and the areas of 2 circles (top and bottom). Again, this is a great method to teach and to learn!!

- *Balancing theory and examples*: Math is a subject learned by doing practice, but don't forget to include some of the theory and reasoning behind the math, or students will turn to simply memorizing formulas without realizing how to properly apply what they have learned. This is where review becomes helpful also, so that students can internalize which concepts are related to which topics.

  You can also teach students about the history of math by, for example, briefly showcasing a Scientist of the Week. Because there is such a variety of thinking styles among students, perhaps teaching it

from a historical point of view will really appeal to some students who otherwise would have been completely disinterested in math. Give some information about the lives of Pythagoras, Newton, Descartes, or other scientists and mathematicians that they may hear about in their theorems and principles. Show students that these are real people who made great contributions to our current understanding of math, and now we need new mathematicians to continue their work. Allow students to do a group project to research a scientist using Encarta or other textbooks and give a short presentation to the class. They will internalize this information because they took ownership of the project and did their own independent learning.

- *I do, we do, you do*: A math teacher must show students how to properly solve problems, but also give them the freedom to take concepts to the next level on their own. A simple example is done as a demonstration for the class, followed by a slightly more complex problem done together, and finally students work on their own to solve problems of varying complexity. This hierarchy, if kept consistent, can develop a sense of trust between teacher and student and can instill a logical progressive mental approach for students.

3.1.6 Teaching towards the test

You may find that there simply isn't enough time in the school year to teach every topic in the syllabus with the time required for student understanding. In addition, from analyzing past exams, you may see that certain topics rarely if ever make their way onto a NECTA exam, and that certain topics are more heavily weighted than others. So, do you skip the less-used topics to save time for the topics that they will actually be tested on, or continue to cover each topic to develop a more thorough math education?

Strategies:

- *Skip the unused topics*: The students (namely Form III's and IV's) are aware of the fact that their entire 4-year education essentially boils down to a couple weeks of NECTA testing at the end of Form IV. They may even encourage you to move on to the topics that are more heavily weighted on the exams to help increase their performance. So you may decide it is in the students' best interest to properly prepare them for their exams by teaching towards the test.

- *Comprehensive learning*: Another option is to forget about the NECTA exam and focus on developing logical minds and a thorough understanding of mathematics within the students. Even if you were to teach strictly towards the exam, there are many factors affecting the results that are outside of your control, such as each student's preparation, the difficulty of the problems given on the exam, and the diligence of the graders in marking the exams (who are often paid by the number of tests they complete and may not have as much of a vested interest in your students' results as you do). So you may find it more beneficial to the students in the long term to try to focus on teaching them a problem-solving approach to use in life, rather than just making sure they get enough points on a test.

3.1.7 USA/TZ different learning methods (GCF, LCM, etc.)

Students learn certain topics in primary school differently than they are typically taught in America. They may have an instilled method of solving problems like finding GCF and LCM of a set of numbers, and if you are not careful, you may end up really confusing them by introducing a method they have never seen before.

Strategies:

- *Ask the students*: Ask your students how they would solve a problem that they may have learned in primary school. Does it match the way you learned it or how the textbook explains it? Giving students a pre-test before each chapter can help you to identify Tanzanian methods of solving problems.

- *Utilize Tanzanian teachers*: Ask a fellow math or science teacher how they learned a topic when they were in school. If necessary, ask for their help in teaching a particular topic using a method that you are not familiar with.

3.1.8 Prepping for NECTA exam

Whether or not you decide to cater your teaching towards the NECTA exam, you can still help students prepare for the test early on. Whichever forms you are teaching, but especially Forms II and IV, which have NECTA exams at the end of the year, you can help the students get ready mentally for their tests throughout the school year so that they are more prepared and less worried come NECTA time.
For more on the format of the math NECTA exam, see the NECTA Manual.

Strategies:

- *Use NECTA format when making exams*: All of your midterm and terminal exams can be written in the same format of NECTA in order to familiarize students with the layout, number of questions, and how to properly manage their time during the exam. Give them less time than they would normally get to challenge their speed as well! They may hate you for making such difficult exams at first, but better to be taken out of their comfort zone earlier rather than being shell-shocked on the actual NECTA exam.

- *Use old test papers for in-class problems*: Before starting a new topic, quickly browse some of the previous years' NECTA exams to see what kind of questions they are using for that topic. If you can get students comfortable with the wording and language of common exam problems while teaching the topic, then why not do it? NECTA exams often repeat problems or use very similar ones from year to year, so exposing students to a particular style of problems from a topic could be a great help to them later on.

- *Offer additional practice exams*: Use a previous year's NECTA exam or create your own to give to interested students as practice. Even the best students may be very comfortable with the math content, but also need to know how to work quickly and efficiently on the exam in order to score an A. Give them a Lightning Round of Section A questions, then Section B, then Section A again. Make it like a sports training drill. Don't just practice until they get it right, practice until they can't get it wrong! This is the level of comfort and confidence students must have in order to succeed on the NECTA exam. Confidence comes from experience!

- *Show students the math NECTA "formula"*: The format of the math NECTA exam is very predictable! Section A topics carry only 3 or 6 points, whereas a single Section B topic (Accounts, Statistics, Linear Programming) can carry 10 marks. Students only have to choose 4 of the 6 questions from Section B, so they should decide which topics they are most comfortable with among these.

  Help students with their time management by breaking down how much time they should spend on each problem. For example, spend 10 minutes on each of the 10 problems from Section A (for 100 minutes), and 20 minutes on each of the 4 Section B problems (for 80 minutes) for a total of 180 minutes, or 3 hours. Then drill them with individual practice questions, giving only 8 minutes for Section A problems and only 15 minutes for Section B problems.

3.1.9 What to do when everyone fails

So now you've put in a ton of effort inside and outside of the classroom for a long time, given practice problems, graded homework, played games, done review, and guess what? They all still failed your exam. Now what? You may not want to show them the results in the fear that they will all become discouraged and hate math. Will you make future tests easier to try and build their confidence, or continue to turn on the heat to challenge them and show them what the NECTA exam will be like?

Strategies:

- *Remain Positive!*: Even if your students did not perform as well as you would have liked on an exam, remember that your attitude towards math strongly influences those of your students. If you get frustrated and give up, how can you expect them to stay motivated? Search for more ways that you can help, what else can you do? Get feedback from the students - what is helpful for them and what isn't? Remember to always praise good performance rather than just punishing the bad, and let the students know when they have done something right. Tanzanian students do not often get positive encouragement at school or at home, so don't forget to give credit wherever it is deserved.

- *Focus on improvement*: Even if the entire class got a zero on your exam, that just means there is more room for improvement. Tell students to each try and increase their scores by 5 points for the next quiz. Start slow and take baby steps. Your successes as a teacher and their successes as students will not be seen overnight.

- *Go easy to build confidence*: Truly, you don't want to discourage students from studying mathematics, so it can be very helpful to start out slow, especially for struggling students, and give them time to develop confidence in a particular topic before moving on to the next. Also, as your teaching style will likely be very different from what they have been used to, it will probably take a while for students to become comfortable in the learning environment that you are creating. But at some point, you will still need to challenge them and take them out of their comfort zones, or their progress will become very limited.

- *Keep it challenging*: It can be greatly beneficial to hold students to high standards of performance and to make them aware of the level of difficulty present in the national exams and the effort required for success. However, this strategy may be best utilized in a gradual way. Giving students reasonable challenges in their math learning over time, and steadily increasing the level of difficulty of your exams may allow students to more easily adapt to your teaching methods and enjoy the challenges you present, rather than become discouraged by them.

3.2 Creating a Positive Math Culture

In addition to the day-to-day teaching challenges faced by volunteer math teachers in Tanzania, it can also be difficult to change pre-conceived negative sentiments towards mathematics that have been instilled in students and teachers alike. Getting students to believe that they can succeed in mathematics will likely prove to be a much greater challenge than simply teaching them the material, and indeed this is a challenge that is felt across Tanzania and even in many developed parts of the world. Provided here are some common challenges and myths perpetuated by a negative math culture, along with some suggested strategies for combating them.

3.2.1 Students don't like math / Math is hard / Ugonjwa wa taifa

You may hear the term "Ugonjwa wa taifa" (National disease) being thrown around by fellow teachers and students regarding the state of mathematics in Tanzania. This pretty accurately sums up the general sentiment held by many Tanzanians that math is difficult and a lost cause for students to learn.

But this is not true!!! There is no disease preventing students in this country from learning math, only a negative attitude and a laundry list of excuses! The best combatant towards a negative attitude is a positive attitude, and it can be just as contagious with the proper inspiration from even a single dedicated individual. Do not allow your students to buy into the idea that they are incapable of doing math. It is this culture of mathematics that must be changed first if any changes in academic performance are to be expected in Tanzania.

Strategies:

- *Math Day*: Choose a Saturday to have Math Day at your school. Promote math, have a local guest speaker, (other teachers at your school) come to talk about the importance of studying math. Show some of the many daily life and career applications of math. Have a school-wide competition using math puzzles and games. Give out prizes and awards. Make math fun for students and show them that you and the rest of the school are not giving up on mathematics!

- *Mathletes/Hisaba-Team*: Start a Mathletes club in your class or for the whole school. Divide students up into teams and give everyone a short 5-question quiz on a given Topic of the Week. The team with the highest average score wins! If you can keep the teams consistent, have weekly head-to-head match-ups among the teams, and keep a running standings board for the whole school to see. The students will enjoy the comparisons to football standings, and will love it even more if they can pick their own team names! Because it is a team effort and every score counts, students will encourage and help each other study for the coming week's topic.

- *Math practicals and projects*: Just because there is no practical component to the NECTA exam, that doesn't mean math isn't a practical subject. Have students build and use a Clinometer to apply principles of trigonometry to find the height of a building. Assign a Statistics project for the class to collect, organize and present data. Stress the use of the Scientific Method.

  As an introduction to statistics, the students can be asked to collect data for the ages of 20 students in school. The students then will organize the data by tallying the results. Then a distribution table can be created with a histogram. This project is a great way to begin Statistics in Form 3. The components of data collection, organizing data in a table and graphing the results in a histogram are made clear in the project.

  Projects can also assess the learning of a topic. Students can be asked to create 3-D objects and to demonstrate the formulas for surface area and volume. Students love to create these 3-D objects! This reinforcement helps the students to maintain formulas and processes to find surface area and volume for the NECTA examination.

- *GAMES!*: Using in-class review games such as Jeopardy are a fantastic way to keep kids excited about math during class and can also provide great feedback leading up to a test.

- *Reward good performance*: Give students an incentive to come to class, do homework, ask for additional practice problems and study for tests by creating a school store and points reward system. For every extra problem they complete, they get one point to be used towards prizes like pens, pencils, erasers, math sets, four figure books, English dictionaries, etc. Get donations from friends and family back home to supply your store! You can even keep a leaderboard of the students with the most points in class to encourage others to catch up.

  Or, if your school has a projector (or even your own laptop computer), offer tickets to Movie Nights on the weekends for well-performing students from that week. Every time a student does something positive, their name goes in the Movie jar. Every time they get in trouble, their name goes in the Kazi (Work) jar. At the end of the week randomly draw names from the jars to see who gets to watch Spiderman and who has to clean your toilet!

  Recognize your bright students by announcing the top performers on each test. Give them stickers and write nice remarks on their papers. Allow them to receive recognition from all of their peers for their hard work. Give regular Student of the Month awards to the best boy and girl at the morning assembly so that the teachers can notice them as well. Once students start to see the positive benefits of studying math, more will follow in their footsteps. Success is contagious!

- *Math Conferences/Seminars*: Many volunteers get involved in regional Boys and Girls Conferences to educate students about HIV/AIDS, malaria, and other health issues. Organize a Math and Science Conference in your region to promote math! Bring your best students and give them a chance to meet and compete with the best students from nearby schools. They will return to your school and tell the other students all about how much fun it was and how they got to see the big city and meet students from other schools. The others will try to work harder so they can go next time.

  It may be difficult to secure funding for this kind of a seminar, since the majority of grant funding tends to come from organizations supporting HIV/AIDS and malaria initiatives (PEPFAR is a major funding source for these conferences). So include these sessions in your conference and relate the topics to math and science! Have kids work on a project to organize and present the statistics on HIV/AIDS and malaria in their communities. What is the probability of a person living with either one? How many 3-D objects can you make out of a bed net to cover your bed at night? Which one uses the least surface area? What is the biology behind these illnesses? Students will learn that they can use math and science to help study and find solutions to these problems in their communities.

3.2.2 Older students have given up on math (Form III, IV)

You may encounter a good number of Form III and IV students who have already checked out of math because they are certain they will fail the math NECTA exam. They may reason that they never learned math throughout primary school or the first few years of secondary school, so why should they start now? It's too late, so might as well just quit now, right?

WRONG! Just because a student has struggled in the past does not mean they should give up. Sure, they were never successful learning math before, but they never had YOU as a math teacher before. You can still help these students to have success in mathematics and gain confidence in themselves.

Strategies:

- *Extra Help*: Many students who give up on math do so because of some particular topic in their past learning that they got hung up on and did not understand very well, and after that they could never keep up with the new material and fell farther and farther behind. For many Tanzanian students, these can be topics such as counting, adding and subtracting. They have the capability to do it, but they've never been able to move at the pace of learning that they need to understand. Spending extra time with your slow learners and showing them that they are capable of doing mathematics when they put forth an effort can make a huge difference in their lives and give them a confidence and sense of accomplishment that they've never known before.
- *Non-math critical thinking and problem-solving*: So maybe a student doesn't enjoy doing math problems, but they can still be a logical and critical thinker! Get them to do puzzles and logic games (see SUDOKU) to improve their critical thinking and problem-solving skills.

3.2.3 Math isn't useful / Don't use math in real life

If you ask students why they don't like to study math, a common response you may get is that they never use math in their lives. Civics and History are practical, they may say, for learning about the government and social practices, but they will never use math, so why do they need to learn it? In order to give students the proper self-motivation to want to learn math, you must show them that they are very, very wrong about this assumption, and in fact math is used in nearly *every* facet of their lives.

Strategies:

- *World of science and technology*: We live in a world of science and technology. Even in Tanzania, nearly everyone has a cell phone, many people use computers and access the internet, and villagers are implementing solar power to provide electricity in rural areas. In order to keep up with the rapid pace at which the world is moving in terms of globalization and advancing technology, it is essential for all students to have at least a basic understanding of math and science.
- *Math in every subject*: Math is used in all other subjects: geography in statistics, civics in population, biology in genetics, physics in vectors, language in counting, chemistry to balance equations... the list is limitless. Therefore, to succeed in other subjects, the student must understand mathematical concepts.
- *Everyday life applications*: Math is used during any money transaction: every time you go to the market or a store, you are using math. A business owner must be proficient in math to ensure she or he is making a profit. Even cooking requires knowledge of the proper ratios of items to mix. If you use these applications in your example problems, it will be easier for students to understand math concepts.
- *Math in many professions*: Math is used in many professions: Tailors use math to make clothes, carpenters use measurements to build furniture, engineers use math to plan roads, shop owners use math to operate his business and to exchange goods for cash, school accountants use math to assess the school budget... Every professional uses math in some area to be successful in his or her career.

3.2.4 Girls can't do math

A common myth in Tanzanian secondary schools is that girls cannot do math as well as boys. This attitude tends to reveal itself through test results, and if you are at a co-ed school, you may see a similar trend. One possible explanation is that when growing up, girls are often given a heavier work load around the house and in the farm than boys, so they may not have the same opportunities for studying or even attending primary school. But it is not acceptable to say that girls are less capable of learning mathematics then boys. With the proper attitude and motivation, any student can excel in math.

Strategies:

- *Girls Conferences*: If your region is holding a Girls Conference, be sure to include some sessions on math and science. Teach girls the importance of learning math and the need for female scientists, mathematicians and engineers in Tanzania and the around the world. Have a female businessperson or accountant or doctor come to speak to the girls to reinforce this point from a successful Tanzanian woman's perspective. Give the girls a forum to play math games, enjoy math, and have the freedom to ask any questions they may not feel comfortable asking at school.
- *Role models*: Teach girls about role models; they should think of someone who they look up to in life and write down all of the characteristics about that person that they want to have themselves. Discuss what characteristics are undesirable in other adults and should not be replicated.

3.2.5 Teachers give up / No motivation for math teachers

After years and years of poor test performance in math by students, other math teachers at your school may become discouraged and claim that there is no motivation for math teachers in Tanzania. Do not let them give up so easily! There are always ways to have a positive impact on the students' lives and make improvements in math.

Strategies:

- *Secondary activities*: Involve your fellow math teachers in secondary activities, such as Math Club, Math Day and Math Conferences and Seminars. Let them see the enjoyment students can get from doing math-related games and activities so they can see the benefits that a teacher can have on students' lives.
- *Recognition for good teachers*: You can also suggest that your school give recognition to good teachers. Have students vote for a Teacher of the Month or Year, based on making an extra effort or trying out alternative teaching methods. You can prepare a certificate to give to these teachers in recognition of their efforts. Again, positive attitudes can be just as contagious as negative ones!

Form I-IV Topics and Activities

4.1 Mathematics Activities for Form I

4.1.1 Numbers (I)

- Flash Cards can be used to illustrate base ten numeration, as well as reinforce simple arithmetic. Use them to play Games such as Around the World to help students master their times tables.
- Use a Number Line to show addition and subtraction of integers. Using students as the number line is a great way to involve them in the lesson. Numbers (I) is a great game for students to practice using a number line.
- To remember the order of operations (BODMAS), create a mnemonic device.

  Ex: Babu, Ondoka! Dereva Mathey Amesema Shusha.
- Operations on integers (BODMAS) can be taught using games such as Four 4's and 24 Squares.
- Be conscious of problem-solving methods common in Tanzanian primary schools for topics such as GCF and LCM which may be instilled in the students.

Target 100

Concept: *Multiplying by a number between 0 and 1 makes numbers smaller. Dividing by a number between 0 and 1 makes numbers bigger.*

Activity: Player 1 chooses a number between 0 and 100. Player 2 has to multiply it by a number to try and get as close as possible to 100. Player 1 then takes the answer and multiplies this by a number to try and get closer to 100. Students take turns doing this, and the student who gets closest to 100 in 10 turns is the winner.

Extensions: Change up the rules and play with division or other numbers such as 0.001 or 1,500. Encourage students to check each other's work by awarding points to the opposing side for catching mistakes.

Toka (Number Line)

Activity: Create a set of Flash Cards with single numbers written on them, ranging from -10 to +10. Be sure to write +'s for positive numbers and -'s for negative numbers. You will also need a Number Line with a range of around -7 to +7.

Start by using only the cards for -5 to +5 and shuffle them. Each player writes his or her name on a small square of paper with tape on the back (post-it notes also work very well). Players post their papers on the 0 as a starting point.

Players take turns randomly drawing a card to see how many spaces they have to move in the positive or negative direction and move their paper accordingly. Have students exaggerate movements on the number line to learn by the hopping method. If a student draws a card that causes them to go off the edge of the number line in either direction, they are out and other students yell, "Toka!"

After the game has continued for a while, add in the higher numbered cards to the draw pile to make things more exciting. The last player remaining on the number line is the winner.

Extensions:

- Use a game board with bottle caps or other items as game pieces to play on a table.
- Substitute Dice, Spinners or other random number-generating tools to see how many spaces must be moved.
- Even Form IV students can have much difficulty adding and subtracting negative numbers. Watch over as students play to make sure they are counting correctly.

4.1.2 Fractions

- See the Hands-on Tools section for how to make Fractions and Decimals.
- Represent fractions using bottles, sticks, colored circles, etc.
- Have a class-wide "Mock Market", where students buy and sell fractions of quantities of items such as rice, flour, oil, milk, etc. Can they adjust the prices according to the fractions purchased?
- See more Games such as Memory Matching, 20 Questions, Guess Who? and Snap!.

Fruit Fractions

Activity: Put 6 pieces of fruit on three tables such that 3 are on one, two on another, and one on the final table. Use the same fruit for all 6, such as bananas or oranges, and make sure each piece is roughly the same size.

Line up 10 students outside the room. Let them in one at a time. Each student must choose to sit at the table where they think they will get the most fruit.

Before the students enter, discuss the following questions with the rest of the class:

- Where do you think they will want to sit?
- How much fruit will each student get?
- If students could change tables during the game, would they?
- Is it best to go first or last?

When all 10 students are seated, ask students to do the following:

- Write down how much fruit each student gets. Write the amount as a fraction and as a decimal.
- Write down the largest amount of fruit any one student gets. Write this amount as a percentage of the total amount of fruit on the tables.

Extensions: Repeat the activity with a different set of students to wait outside the room. Try with a different number of tables, a different number of pieces of fruit or a different number of students.

4.1.3 Decimals and Percentages

- See the Hands-on Tools section for how to make Fractions and Decimals.
- Create movable numbers on manila paper to show that decimal points must be lined up for addition and subtraction.
- Play a decimals and percentages BINGO game. The students fill their cards with numbers in one form (e.g. decimals) and the teacher calls out numbers in another form (e.g. percentages).

Students must first convert the number before being able to cover spaces on their boards. Play this game with fractions as well.

- Have students calculate their percentage on homework or in-class assignments, based on the number of questions and how many were answered correctly.
- See more Games such as Memory Matching, 20 Questions, Guess Who? and Snap!.

4.1.4 Units

- Use tape measures, rulers and metre rulers to show conversions in length.
- Use various sized bottles to demonstrate capacity. Show that three 500 mL bottles is the same volume as a single 1.5 L bottle to reinforce multiplication of fractions.
- Students can create a clock to learn units of time.

Unit Scavenger Hunt

Activity: Split class into groups and have them hunt around the classroom to measure different objects (desks, tables in the lab, a body part such as head or arm length).

Extensions: Repeat this activity with objects suitable for finding volume or masses

4.1.5 Approximations

- Flash Cards can be used to create movable numbers for teaching significant figures.
- Have students guess how many pieces of candy are in a small jar. The student who gets closest to the correct number wins the candy!
- Estimate the number of students in each stream of every form to approximate the total students at school. Compare this to the actual total. Is it a good estimate?
- Ask students to approximate how much corn can be yielded from an acre of farmland. How many acres would be needed for a certain desired profit?

Rounding Staircase

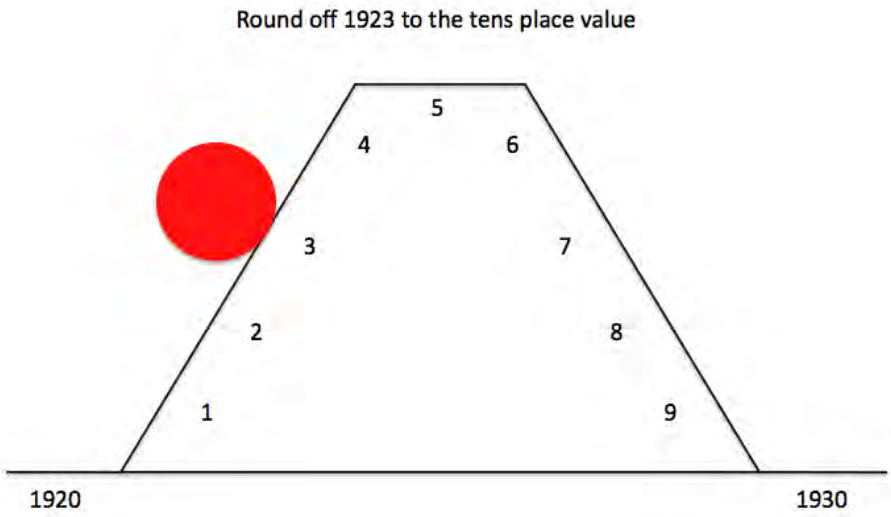
Concept: *Teach students how to round off numbers to a given decimal place, significant figure, or place value.*

Activity: Teacher writes a number on the board and gives the students a place value for students to round to. One student comes up to the front of the class and uses the staircase to determine the correct number.

To use the staircase, the student will put the ball on the correct digit that is to the right of the desired place value. If the digit is 1, 2, 3, or 4, the ball will roll to the lower value. If the digit is 6, 7, 8, or 9, the ball will roll to the higher value. If the digit is 5, the ball will be placed at the top of the staircase. The teacher will then need to ask a further question: *Is the digit in the desired place value even or odd?* The answer will determine which way the ball should roll.

Extensions: Repeat the activity for given decimal places or number of significant figures.

Example:



4.1.6 Geometry

- Use a Geoboard to create lines and various polygons and angles.
- Circles can be drawn on the ground using a string radius. Students can construct their own circles by making their own Compass.
- Students can create their own Protractor to construct various angles.
- Salama says: students act out points, lines and angles using their arms.
- Cut out a large triangle having any dimensions. Label each angle and then cut out and tape them together on the board to show that they form a straight line and thus add up to 180°. Repeat for different polygons to discover formulas for finding interior and exterior angles.
- See Games such as Around the World and Battleship.

Estimating Angles

Concept: *Angle is a measure of turn. It is measured in degrees. Angles can be acute (less than 90°), right (90°), obtuse (greater than 90°), or reflex (greater than 180°).*

Activity A: Player 1 chooses an angle, e.g. 49°. Player 2 has to show that angle without using a protractor. Player 1 measures the angle with a Protractor. Player 2 is given points equal to the difference between the angle drawn and the intended one. For example, Player 2's angle is measured to be 39°, so Player 2 scores 10 points (49° - 39°). Students take turns naming and drawing angles. The winner is the player with the lowest score.

Activity B: Each player draws 15 angles on a blank sheet of paper. They swap papers and estimate the size of each angle. Then they measure the angles with a Protractor and compare the estimate to the exact measurement of the angles. Points are scored based on the difference between the estimate and the actual size of each angle. The player with the lowest score wins.

Tessellation Investigation

Concept: *A tessellation is a repeating pattern of one shape in more than one direction without any gaps. A semi-regular tessellation is a repeating pattern of two shapes in more than one direction without any gaps.*

A regular shape will tessellate if the interior angle is a factor of 360°. Semi-regular tessellations work if the sum of a combination of the interior angles of the two shapes is 360°.

Activity: Give students a collection of regular polygons. Ask them to find out:

- Which polygons can be used on their own to cover a surface without any gaps?
- Which two polygons can be used together to cover the surface without any gaps?
- Explain why some shapes tessellate on their own and others tessellate with a second shape.

Slap the Board

Activity: Draw different shapes and angles on the board. Split your class into two teams and make a large aisle in the middle of the classroom. Call one student from each group to stand in the back of the classroom. Then call out a shape or an angle and the student runs to the board. Whomever slaps the shape first wins a point for their team.

Extensions: Great review game that can be used for a variety of topics.

4.1.7 Algebra

- Bring a bucket of apples and bananas. Apples represent the "a" and bananas represent "b". Unlike objects cannot be added.
- Colored chalk can be used. Red and blue cannot be added together.
- Use a scale to represent both sides of the equation. If something is added or subtracted to one side, it must be done the same to the other side to maintain balance.
- Use cards by writing the variable on one side and its value on the other. When the equation has been solved, the variable reveals its identity by flipping over.

Inverse Operations

Concept: *Addition is the inverse of subtraction and subtraction is the inverse of addition. If you do an operation followed by its inverse, you arrive where you started (e.g.* 7 + 2 - 2 = 7*).*

When you are dealing with more than one operation, you arrive where you started if you do the inverse of each operation in the opposite order. For example:

$$7 + 2 = 9$$
$$9 \times 3 = 27$$

then to reverse: $27 \div 3 = 9$, $9 - 2 = 7$.

Activity: Give students instructions such as:

- I am thinking of a number. I multiply it by 5 then subtract 7. The answer is 58. What number was I thinking of?
- I'm thinking of a number. I multiply it by 3. I then subtract 6. I then divide by 2 and then add 5. The answer is 23. What was my number.

Extensions:

- Have students discuss strategies for finding the original number.
- Get students to find inverses of equations in two variables by working backwards.
- Use more advanced operations such as exponents, radicals and logarithms.

Simultaneous Equations

Activity: Write an equation at the top of the board, e.g. $x + y = 10$. Divide the rest of the board into two columns. Ask each student to do the following:

- Think of one set of values for x and y which make the equation on the board true. Do not tell anyone these values.
- Make up another equation in x and y using your values.
- Invite students one by one to say the equations they have made up. If their equation works with the same values as the teacher's equation, write it in the left hand column. If it does not work, write it in the right hand column.

Ask students to:

- Work out the values of x and y for each set of equations.
- Discuss the methods they used to solve each set of simultaneous equations.

Study the two lists of equations on the board.

- Are any pairs the same?
- Can any of the equations be obtained from one or two others?

4.1.8 Numbers (II)

- A Number Line is the best tool for teaching numbers, even fractions that are rational numbers.

4.1.9 Ratio, Profit and Loss

- Ratio can be taught with objects to show that Juma gets 2 items for every 3 items that Mary receives.
- Show profit and loss through a bottle cap business demonstration, for a given buying price and selling price per bottle cap.
- Have students investigate ratios of different body parts (see activity below).

Body Part Ratios

Concept: *Ratio is the comparison of two quantities or measurements. Ratios can be written as follows: a:b, age:height, 2:3. Ratios show how many times larger or smaller one thing is compared to another.*

Activity: Make a list of body parts that can be measured with a piece of string, such as:

- circumference of the wrist
- circumference of the neck
- circumference of the base of the thumb
- circumference of the waist
- distance from shoulder to finger tip
- height
- circumference of the head

Cut a length of string the same length as each body part in the list. Find the ratio of things like:

- wrist:neck
- waist:height

Extensions: Investigate other body ratios. Record your findings by using the thumb as a reference value of 1. Find other ratios such as:

- nose length:thumb length

4.1.10 Coordinate Geometry

- A Geoboard can be used to demonstrate slope, midpoint, parallel and perpendicular lines, etc.
- Allow students to create their own Cartesian Plane out of old calendars, seed bags or whatever they choose.
- Use Dice to generate random coordinates for practice with plotting points and to make homework or in-class problems on slope, midpoint, distance formula, etc. Make four dice out of manila paper (2 red - one of +'s and -'s, and one of the numbers 1-6, and then 2 blue in the same way). Students roll the dice to find an x-coordinate (blue) and a y-coordinate (red) and then must plot the point on the Cartesian plane.
- Play a Battleship game to help students read and plot points.

Coordinate Quadrilaterals

Concept: *Coordinate pairs give the position of a point on a grid. The coordinate pair (2,3) describes a point with a horizontal displacement of 2 and a vertical displacement of 3 from the origin.*

Activity: Draw a large pair of axes on the ground or on a large piece of card on the ground. Label the x and y axes.

Place 4 bottle caps on the grid as the vertices (corners) of a quadrilateral. Record the 4 coordinate pairs. Make other quadrilaterals and record their coordinate pairs.

Sort the quadrilaterals into the following categories: square, rectangle, rhombus, parallelogram, kite, trapezium. In each category look for similarities between the sets of coordinate pairs.

4.1.11 Perimeter and Area

- Use a Geoboard to help students discover formulas of area and perimeter for squares and rectangles.
- Using graph paper, have students cut out rectangles having an area of 48 square units. How many different sets of dimensions be used to generate this area?
- Circles, such as bike wheels, can be used to show circumference and also to measure the perimeter around the school or a classroom.
- Use string to measure the circumference and diameter of a circle, and to derive π.
- Gather several small circles or cylindrical items of various sizes. Trace the circular outlines on graph paper and for each one, count the number of squares that make up the area and the number of units of length of the radius. Tabulate the results. Can you discover a relationship between radius and area for different sized circles?

4.2 Mathematics Activities for Form II

4.2.1 Exponents and Radicals

- Use Flash Cards with "x" written on them to show that multiplying four of these is equal to "x" to the fourth power.
- Add and remove cards in the numerator and denominator to demonstrate the laws of exponents in this way.
- Guide students to make tables or posters of the laws of exponents to be posted around the classroom.

4.2.2 Algebra

- Use colored chalk to teach BODMAS.
- Factor trees can be helpful when teaching factorization.

4.2.3 Quadratic Equations

- Use crazy names for variables so that students do not combine different variables.
- Use a song to help students remember the quadratic formula.

  Here are two examples of songs that can be used:

Pop! Goes the Weasel

Original Song	Quadratic Equation Lyrics
All around the mulberry bush	x equals the opposite b
The monkey chased the weasel	plus or minus square root
The monkey thought 'twas all in good fun	B squared minus 4 a c
Pop! Goes the weasel	all over 2 a

Row, Row, Row your Boat

Original Song	Quadratic Equation Lyrics
Row, row, row your boat	x equals minus b
gently down the stream	plus or minus square root
merrily, merrily, merrily, merrily	b squared minus 4 a c
life is but a dream	divided by 2 a

4.2.4 Logarithms

- Help students create charts of the laws of logarithms. These can be hung around the classroom for easy reference.
- Break into 4 groups and have each group explain the 4 laws of logarithms with examples for all of the logarithms
- Ensure students get some practice using four figure tables to solve problems as this is often required on the NECTA exams.

4.2.5 Congruence

- Draw pairs of various congruent shapes on cards and play Games such as Memory Matching and Snap! to get students to identify congruent pairs.
- Use origami paper to show how folding squares can produce different congruent shapes.

4.2.6 Similarity

- Use Games such as Memory Matching and Snap! to help students identify and pair off similar figures.
- Construct a 2-D house out of cut out triangles and rectangles. Then have students cut out a set of shapes that are similar to the ones you used, and another set which are not similar. Which house better resembles the one you made?

4.2.7 Geometrical Transformations

- Use a Geoboard or an x-y plane in the ground to show rotation, reflections, etc.
- Students can be used to demonstrate rotation, reflection and translation.
- Use bottle caps or a student's name to show original and transformed shapes side-by-side.
- Mirrors can be useful in showing students reflections.

Bottle Cap Transformations

Concept: *Transformations are about moving and changing shapes using a specified rule. Four ways of transforming shapes are reflection, rotation, enlargement and translation.*

Activity: (Reflection) Every point has an image point at the same distance on the opposite side of the mirror line.

- Place 4 bottle caps, top-side up, to make a quadrilateral. Record the coordinate pairs. Place another 4 bottle caps, teeth-side up, to show the mirror image of the first quadrilateral reflected in the line $y = 0$. Record these coordinate pairs and compare to those of the original.
- Show different quadrilaterals reflected in the line $y = 0$. Note the coordinates and investigate how the sets of coordinates are related.
- Make reflections of quadrilaterals in other lines such as $x = 0$ and $y = x$.

Activity: (Rotation) All points move the same angle around the center of rotation.

- Place bottle caps, top-side up, to make a shape. Record the coordinates of the corners of the shape. Place another set of bottle caps, teeth-side up, to show the image of the shape when it has been rotated 90° clockwise about the origin. Record these new coordinates and compare to the originals.
- Repeat for different shapes, recording the coordinates before and after rotation and investigating how they are related.
- Try rotations of other angles like 180° clockwise or 90° anticlockwise.

Activity: (Enlargement) A shape is enlarged by a scale factor which tells you how many times larger each line of the new shape must be.

- Place bottle caps, top-side up, to make a shape. Record the coordinates of the corners of the shape. Place another set of bottle caps, teeth-side up, to show the image of the shape when it has been enlarged by a factor of 2. Record the new coordinates and compare to the originals.
- Show different shapes enlarged by scale factors such as 5, $\frac{1}{2}$ and -2.

Activity: (Translation)

All points of a shape slide the same distance and direction.

- Place bottle caps, top-side up, to make a shape. Record the coordinates of its corners. Place another set of bottle caps, teeth-side up, to show the image of the shape when it has been translated left 4 units and record its new coordinates. Compare the two sets of coordinate pairs and investigate how they are related.
- Now try different translations and try to predict the coordinates of the image.

4.2.8 Pythagoras Theorem

- Help students derive Pythagoras Theorem by using a right triangle made with squares acting as each side. Use the areas of the squares to show that $a^2 + b^2 = c^2$.
- Demonstrate a proof of Pythagoras Theorem using a square sheet of paper folded so that each side is split into lengths of a and b, creating an inner square of side length c.

4.2.9 Trigonometry

- Guide students to develop tables of values of trig ratios for special angles for $-360° \leq \theta \leq 360°$ and then use them to graph sin, cos and tan functions.
- Play a BINGO game using trig ratios for some special angles. Instead of B-I-N-G-O, column headers are sin, cos and tan. Students fill their cards with various values under each trig function, and the teacher calls out a particular trig function and angle. For example, the teacher calls, "$\sin 30°$" and students having a "0.5" under their sin column get to cover that space.
- Have students create a Clinometer to see the application of trigonometry in determining the height of a building or tall tree.

4.2.10 Sets

- Construct a tangible Venn diagram to place movable shapes, colored objects, etc.
- Demonstrate sets and subsets using playing cards, Dominoes, dinner ware or even students themselves!

4.2.11 Statistics

- Have students collect data on family or fellow students based on age, height, school level, children in the family, etc. They then organize their data in bar graphs, line graphs, pie charts, histograms, frequency distribution tables, etc. Students can also find mean, median and mode by using formulas or approximations from their graphs and tables. This is a great class project or math practical and can also be used as a way to teach the Scientific Method.
- Incorporate statistics on HIV/AIDS and malaria to increase awareness of the students in their communities.
- Use current football stats on well-known teams to excite and engage students.
- Arrange students in a line according to height or age to demonstrate median.

4.3 Mathematics Activities for Form III

4.3.1 Relations

- Use Dice and Dominoes for forming Domain and Range sets.
- Use a Geoboard or Cartesian Plane to graph relations.

4.3.2 Functions

- Teach functions using the "Function Jiko" method:
  - Functions can be likened to cooking ugali on a charcoal jiko. The necessary inputs are added to the jiko - these can be grouped together to represent the independent variable, x (i.e. unga and water).
  - Then "stuff happens" to the input - these are the operations on the independent variable such as multiplication, addition, etc. (i.e. stirring and heating). These must be performed in the correct order to receive the desired product.
  - Finally, the function "poops out" its output after all necessary operations are performed - this is the dependent variable, y (i.e. ugali).
  - Use a real pot, spoon, etc. to demonstrate this concept to the class. Stress the fact that if you change the input, the output is affected accordingly. In a function, every input correlates to exactly one output.
- Graph functions on a Geoboard or Cartesian Plane.
- Play a Battleship game to reinforce graphing different functions.

Discover the Function

Activity: Think of a simple function, e.g. $x \times 3$. Write a number on the left side of the chalkboard. This will be the IN number, though it is important not to tell students at this point. Opposite your number, write the OUT number. For example:

$$10|30$$

Show two more lines. Choose any numbers and apply the same function rule.

$$5|15$$
$$7|21$$

Now write an IN number only and invite a student to come to the board to write the OUT number.

$$11|?$$

When students show that they know the rule, help them to find the algebraic rule. Write x under the IN column and invite students to fill in the OUT column.

$$x|?$$

When students have shown that they know the function, move on to another.

Extensions:

- Try functions with two operations, squares, cubes, radicals, etc.
- Challenge students to find functions with two operations which produce the same table of IN and OUT values.
- Challenge students to show that the function $x \times 2 + 2$ is the same as $(x + 1) \times 2$.
- How many other pairs of functions can they find that are the same?

4.3.3 Statistics

- Have students collect data on family or fellow students based on age, height, school level, children in the family, etc. They then organize their data in bar graphs, line graphs, pie charts, histograms, frequency distribution tables, etc. Students can also find mean, median and mode by using formulas or approximations from their graphs and tables. This is a great class project or math practical and

can also be used as a way to teach the Scientific Method.

- Incorporate statistics on HIV/AIDS and malaria to increase awareness of the students in their communities.
- Use current football stats on well-known teams to excite and engage students.
- Arrange students in a line according to height or age to demonstrate median.

4.3.4 Rates and Variations

- Demonstrate 2:3:5 using blocks, sticks, bottles, etc. so that it will be concrete learning.
- Use different sized syringes or tubes to show that the rate of water flow is proportional to the size of the tube opening.
- Use practical examples of direct variation, such as opening and closing a faucet (bomba), cooking ugali and studying vs. test performance. Examples of indirect variation include time spent talking on the phone vs. remaining vocha, notes written vs. chalk length, etc.
- Have students sketch graphs of common practical examples of variations without using numbers.
- Incorporate formulas from science subjects, such as chemical reaction rates, Ohm's Law, etc.
- Ask students to come up with examples on their own to demonstrate comprehension.

4.3.5 Sequences and Series

- Translate word statements into math expressions (e.g. 4th term is 10 more than the 2nd term).
- List many numbers in a sequence to prove the sum formulas.
- Use a Jeopardy-style game to challenge students' pattern recognition using numerical and geometrical sequences. This is a great tool for developing logical thought processes.
- Make colored posters to hang around the classroom highlighting the distinctions between arithmetical and geometrical progressions.
- Have students act out sequence patterns.

Sequence Steppers

Concept: *A number sequence has a starting point and a step size. For example, starting at 3 and going up in 5's produces the sequence:*

$$3, 8, 13, 23, \ldots$$

The Fibonacci sequence is made by starting with the digits 1 and 1. Each new term is made by adding together the previous two terms. That is,

$$1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

Activity: Have students make or imagine a number line stretching on both sides of them. Tell them to locate 0. Tell the students they are now going to go for walks along their number lines. Give them instructions such as:

- Start at 0, step on all multiples of 3. How many steps before you pass 50?
- Start at 4 and go up in 7's. Will you land on 100?
- Start at 5 and go down in 11's. How many steps before you pass -100?
- Start at 9 and go up in a Fibonacci sequence. How many prime numbers do you land on before you get to 100? What are they?
- Start at 7 and go up in 4's. As you land on each number, look at the units digit. When do they start repeating? How long is the cycle?
- Start at -5 and go down in 3's. As you land on each number, look at the units digit. What is the pattern?
- Start at 0 and walk along the line until you get to 10. Now fold your line around so that 11 ends up next to 9. Look at the other pairs you have created. What is 0 next to? What do you notice about these pairs of numbers?

4.3.6 Circles

- See the Hands-On Tools section for how to make Circles.
- Play a game to have students stick paper labels to corresponding properties of circles.
- Use a clock to sweep out a span of time to represent a sector of a circle.
- Cut out circles to draw on for proving circle theorems in class.
- Have students draw what each theorem states to reinforce the concept.

4.3.7 Earth as a Sphere

- Have students make spheres out of bamboo strips. See more in Hands-On Tools for making a Globe.
- Use globes, if available, or else footballs, basketballs or volleyballs to show students longitude and latitude.

4.3.8 Accounts

- Teach the principle of double entry using the "Accounts Stendi" method:
  - Each account is like a bus stand, having an entrance (Dr.) side and an exit (Cr.) side.
  - When a transaction occurs between two accounts, or bus stands, the money must be transported via the Transaction Express.
  - The bus exits the first stendi, but the driver must "sign out" before leaving by writing the Date of the transaction (Date of travel), Particulars (the bus's destination city/account), Folio (the corresponding number code for the destination) and Amount (the money being delivered) under the Cr. side (exit) of the departure stendi.
  - When the bus reaches its destination, the driver must now "sign in" by writing the Date, Particulars (city/account of origin), Folio (number code for the origin) and Amount under the Dr. side (entrance) of the arrival stendi.
  - This analogy helps students to see why it is necessary to record each transaction twice in the account books. For example, a Purchase must be recorded for two purposes: to account for the payment of money *from the Cash Account*, and to account for the receipt of goods *into the Purchases Account*.
  - Make real accounts and a real Transaction Express bus out of manila paper to demonstrate this concept.

- Use movable strips of paper as transactions and allow students to come up to the board to place them in the proper accounts.
- Have a small duka demonstration with the class. You have a tomato or bottle cap business and need to keep track of purchases, sales, expenses, such as transport and duka monthly rent fees, and want to know if you are making or losing money.
- See more in Hands-On Tools for making Accounts.

4.4 Mathematics Activities for Form IV

4.4.1 Coordinate Geometry

- A Geoboard can be used to demonstrate slope, midpoint, parallel and perpendicular lines, etc.
- Allow students to create their own Cartesian Plane out of old calendars, seed bags or whatever they choose.
- Use Dice to generate random coordinates for practice with plotting points and to make homework or in-class problems on slope, midpoint, distance formula, etc. Make four dice out of manila paper (2 red - one of +'s and -'s, and one of the numbers 1-6, and then 2 blue in the same way). Students roll the dice to find an x-coordinate (blue) and a y-coordinate (red) and then must plot the point on the Cartesian plane.
- Help students use Pythagoras Theorem to derive the distance formula.
- Play a Battleship game to help students read and plot points.
- Have students create different shapes on a coordinate plane by plotting bottle caps.

4.4.2 Area and Perimeter

- Use a Geoboard to help students discover formulas of area and perimeter for squares and rectangles.
- Using graph paper, have students cut out rectangles having an area of 48 square units. How many different sets of dimensions be used to generate this area?
- Circles, such as bike wheels, can be used to show circumference and also to measure the perimeter around the school or a classroom. Alternatively you can use plates, chai cups, balls, coins, water bottles, etc.
- Use string to measure the circumference and diameter of a circle, and to derive π.
- Gather several small circles or cylindrical items of various sizes. Trace the circular outlines on graph paper and for each one, count the number of squares that make up the area and the number of units of length of the radius. Tabulate the results. Can you discover a relationship between radius and area for different sized circles?
- Students make various polygons using a Geoboard or cutting them out of paper, and then use the shapes to discover formulas.
- Have students draw polygons on graph paper (or create their own graph paper) to discover relations between areas of similar figures.

4.4.3 Three-Dimensional Figures

- See the Hands-On Tools section for how to make Three-Dimensional Objects.
- Have a challenge to see who can identify the most 3-dimensional objects in the classroom.
- Use hollow containers or objects that can be opened to see interior angles and planes.
- Do a design challenge with students.

3-D Design Challenge

Concept: *Volume is the amount of space a solid takes up. It may be found by counting cubes or by calculations for regular solids.*

Surface Area is the area of the net of a solid. It can be found by counting squares or by calculation for regular shapes.

Activity: Students must construct 3-D shapes given certain design constraints. For example:

- You may only use 1 sheet of paper. What is the largest volume cuboid you can make?
- You need to make a box which has a volume of 96 cm^3. The box can be any shape. What is the smallest amount of card you need?
- You have a 24 cm $\times$ 24 cm square of card. You can make a box by cutting squares out of the corners and folding the sides up. Make a box with the largest volume. What is the length of the side of the cut-out squares? Try for other sizes of square cards or with rectangular cards.
- You have a rectangular piece of card which is 24 cm $\times$ 8 cm. What is the largest volume cylinder you can make?
- You are going to make a cylinder which must have a volume of 80 cm^3. What is the smallest amount of card you need?

4.4.4 Probability

- Use Dice, playing cards, rock-paper-scissors, coins, bottle caps, Flash Cards, Spinners, fruits or any objects to demonstrate probability.
- Have students determine the probability of selecting a boy from the class, or two boys. Does the result change if the first one goes outside before the next selection is made?
- Relate to genetics in Biology, or incorporate data on HIV/AIDS and malaria to increase awareness.
- Use a probability line to teach key probability terms and to apply probability to real-life situations.

Feely Bag

Activity: Put different colored beads (or bottle caps) in a bag, e.g. 5 red, 3 black and 1 yellow bead. Invite one student to take out a bead. The student should show the bead to the class and they should note its color. The student then puts the bead back in the bag. Repeat this many times. Stop when students can say with confidence how many beads of each color are in the bag.

Probability Line

Concept: *Probability is the likelihood of an event happening. To describe the likelihood of an event happening, we use words like: very likely, unlikely, certainly, even, impossible, probable, very unlikely, no chance, definite, dead certain, etc.*

Activity: Tie a piece of string to make a straight line across the classroom. Peg cards with 0 and 1 written on them to either end of the line. This is a probability line that goes from 0 (impossible) to 1 (certain). Using clothes pins, peg cards on the line to show the likelihood of different future events. Make up events of your own and put them on cards on the line. Examples of events include:

- It will rain tomorrow.
- I will go to school tomorrow.
- I will throw a 6 on the die.

Extensions:

- Discuss where different word descriptions should be placed on the probability line: even, very likely, good chance, dead certain, possible, unlikely, no chance, etc.
- Have students write events corresponding to the aforementioned descriptions. (e.g. for "dead certain," Mr. Jack will eat ugali next week.)
- Mutually exclusive events - Have students compare the probabilities of mutually exclusive events in terms of where they lie on the clothesline and how to describe them using words. (e.g. What is the probability that Kikwete will be re-elected? What is the probability that he will not be re-elected?)
- Dependent events - Give students pairs of dependent events A and B. Where is B located on the clothesline given that A has happened? Where is B located given that A has not happened? (e.g. If you pass math class, what is the probability that you'll pass the NECTA exam? Alternatively, if you fail, what is the probability that you'll pass the NECTA exam?)

Curious Combinations

Concept: *All possible outcomes can be listed and counted in a systematic way.*

Activity: How many ways can you arrange three different bottle caps in a line? Investigate for different numbers of bottle caps. Can you develop a relation between the number of items and the number of possible outcomes?

Left and Right

Activity: Make a number line or board with 0 at the center and 5 evenly spaced numbers on either side. A player sits at each end. Use a bottle cap as a counter and start it at 0.

- Player 1 rolls 2 dice. Find the difference between the two numbers showing.
- If the difference is 0, 1 or 2, move the counter one space to the left.
- If the difference is 3, 4 or 5, move one space to the right.
- Take turns rolling the dice, calculating the difference and moving the counter. Keep a tally of how many times the right player wins and how many times the left player wins.
- Collect the results of all the games in the class. What do you notice about the results?
- Is the game fair? Why or why not?
- Challenge students to redesign the game so that the chances of winning are:
  - better than losing
  - worse than losing
  - the same as losing

4.4.5 Trigonometry

- Guide students to develop tables of values of trig ratios for special angles for $-360° \leq \theta \leq 360°$ and then use them to graph sin, cos and tan functions.
- Play a BINGO game using trig ratios for some special angles. Instead of B-I-N-G-O, column headers are sin, cos and tan. Students fill their cards with various values under each trig function, and the teacher calls out a particular trig function and angle. For example, the teacher calls, "$\sin 30°$" and students having a "0.5" under their sin column get to cover that space.
- Have students create a Clinometer to see the application of trigonometry in determining the height of a building or tall tree.

4.4.6 Vectors

- Use colored origami paper cranes or cut out airplanes to use as points on a Cartesian Plane. The birds or planes start at one point and travel in a specified direction towards the destination point.
- Relate to applications in Physics.
- Use colored chalk to show that unlike components cannot be combined.

4.4.7 Matrices and Transformations

- Charts and colored chalk must be used to demonstrate operations on matrices.
- Attach a string to the origin and stretch it out to an object to show rotation about the origin.
- Use mirrors or spoons to show reflection.
- Use bottle caps to show original and transformed shapes side-by-side.

4.4.8 Linear Programming

- A Geoboard can be used to demonstrate maximum and minimum.
- Drill students on how to generate constraints based on the word problems. Use tables to organize information. This requires the most practice as students are not familiar with many English terms.
- Make a math vocab list of key terms such as "at least," "at most," "no fewer than," "not more than," "minimum" and "maximum."

Games and Puzzles for Stimulating Interest in Mathematics

Math must be stimulated by puzzles, games, patterns and other forms of hands-on and interactive learning. An interest in mathematical puzzles generates creative thinking and motivates students in a way that standard textbooks can rarely achieve. Given that the vast majority of Tanzanian students do not have access to any textbooks at all, the importance of interactive learning techniques such as those provided in this section must not be underestimated.

Many of these games and puzzles reinforce very basic mathematical concepts, so they are a great way to help solidify students' understanding of such crucial topics through a variety of approaches. Given some creative dimension to their learning, students become more enthusiastic about practicing their math skills.

5.1 Games

5.1.1 Jeopardy

Applicable Topics: Any topic as a review game
Number of Players: 3-6 Groups

How to Play:
Divide the class into equal groups and allow them to select their own team name. Designate one student from each team as the writer. Select 5 categories (see suggestions below) and write them at the top of the board on the far left. Beneath each category, write 100, 200 and 300, using colored chalk if possible. Each question's point value is based on its difficulty (100 - easy, 200 - average, 300 - difficult). Write a column for each team on the right side of the board and leave the center of the board empty.

The first team selects a question by stating a category and a point value. The teacher writes the problem in the center of the board for all groups to see. Groups work together to solve the problem, but only the designated writer may present the answer. When an answer is found, writers must hold their notebooks high above their heads so that no further changes can be made, and everyone in the group must remain silent. The first group to finish with a correct answer gets the points and may select the next question. However, groups only get **one** attempt at each question, so if they are wrong, they are finished until the next question. If none of the groups answer correctly, the teacher gets the points! Once a correct answer has been given, have one student come up to the board to show the rest of the class.

This is a wonderful review game to play before an exam, since it also shows students what topics they need to study.

Extensions:
JEOPARDY works great as a review game for any topics covered in class. However, it can also be useful for teaching logical thinking skills, and may be incorporated as part of a Math Day-type promotional school event, or a Math and Science Conference or Competition. Here are some suggested categories for such game variants.

- *Pattern Problems* - pattern recognition sequences for progressions of shapes, rotating figures, letters, etc. Give the first 3, students must find the following 2. (e.g. Abcd, eFgh, ijKl, ___, ___)

- *Sequence Solvers* - Numerical sequences or series with various rules. Give the first 4, students must find the following 2. (e.g. 1, $\frac{1}{22}$, 333, $\frac{1}{4{,}444}$, ---, ---)

- *Creative Classes* - Write 9 items on the board, mixed together randomly. Groups must find common characteristics among the items to classify them into 3 groups of 3. (e.g. Oxygen, Milk, Water, Pen, Table, Carbon Dioxide, Oil, Air, Rice - *Gases, Liquids, Solids*)

- *Word Work* - Give students a word that applies to math or science. Groups must make at least four 3-or-more-letter words by re-arranging the letters. (e.g. MATHEMATICS - math, mat, the, them, ...)
- *Design Dilemmas* - Groups must solve design problems using given constraints. For example:
  - You must design a rectangular building having a volume of 64 m^3. What is the smallest surface area you can use?
  - You need to fence a farm having an area of 48 m^2. What is the smallest length of fence you can use to border the full area of the farm?

5.1.2 Memory Matching

Applicable Topics: Fractions, Decimals and Percentages, Congruence, Similarity, Area and Perimeter, Algebra
Number of Players: 2-6

How to Play:
Prepare roughly 10 pairs of cards, with each set being a different pair of congruent shapes. Mix all cards together and place them in a grid, face down. Players take turns turning over 2 cards at a time. If the pair is a match of congruent shapes, they get to keep them and try another pair of cards. If they are not a match, they must turn them face down again, in their same places, and it is the next player's turn to try. The game ends when all cards have been paired off. The player with the most pairs wins.

Extensions:

- Use pairs of similar shapes instead of congruent shapes.
- Make pairs of equivalent fractions, decimals or percentages. All students work together to determine if a given pair are equivalent.
- Make pairs with one card being a math equation and its pair being the solution. This can be applied to nearly any topic.
- Turn the game into a Concentration game. On a big sheet of paper the size of the full grid of cards, write down a math problem, e.g. a pair of simultaneous equations or a problem on area or perimeter. Without students seeing this sheet, cover it with the face down card grid. As students remove successfully paired cards, the picture beneath is revealed, piece by piece. The first student to solve the problem underneath is the winner. They may need to make inferences or assumptions while parts of the board are still covered.

5.1.3 Around the World

Applicable Topics: Numbers (I), Fractions, Decimals and Percentages, Geometry, Area and Perimeter
Number of Players: Entire class

How to Play:
Use Flash Cards to drill basic math operations. The first 2 students stand up next to each other at their desks. The teacher holds up a card, e.g. 5×2. The first students to give the correct answer moves on, the other student sits down. The winner then faces the next student in order, who stands up and another card is drawn. When a player is defeated, he or she must take the seat of the student who beat them. See who can make it the farthest without being beaten. Can anyone make it around the entire classroom and back to their original seat? It is important for the rest of the class to be silent while two players are facing off.

Extensions:

- Instead of operations, have students compete in converting fractions, decimals and percentages.
- Show students simple shapes with dimensions and have them state the area or perimeter.
- Show students various shapes and they must be first to identify the shape, e.g. pentagon, rhombus, isosceles triangle, etc.

5.1.4 BINGO

Applicable Topics: Numbers (I), Fractions, Decimals and Percentages, Trigonometry
Number of Players: Entire class

How to Play:
Each student makes a 4×4 or 5×5 table in their notebook. Students then fill their game card with random numbers from a given range specified by the teacher, e.g. from 1-40, even numbers from 1-100, etc. The teacher then calls out numbers one at a time. If students have that number on their cards, they place a marker (small square of paper) to cover it up. When a player has a full row, column or diagonal covered, they must shout, "Bingo!" and the teacher checks to confirm all of the numbers were called. Players then clear their cards and a new game begins.

Extensions:

- Instead of just calling the numbers, give students a short problem that they must solve first. For example, instead of reading "5," write on the board the equation, "4×3 - 7" Use this method to review BODMAS.
- Drill conversions among fractions, decimals and percentages. Students fill their cards by writing one form (e.g. decimals), but the teacher reads numbers in another form (e.g. fractions). Be sure to tell students what range to select numbers from when filling up their cards.
- Apply to trigonometry. Instead of B-I-N-G-O, column headers are sin, cos and tan. Students fill their cards with various values under each trig function, and the teacher calls out a particular trig function and angle. For example, the teacher calls, "$\sin 30°$" and students having a "0.5" under their sin column get to cover that space.

5.1.5 Battleship

Applicable Topics: Coordinate Geometry, Geometry, Relations, Functions
Number of Players: 2

How to Play:
Each player makes 2 coordinate planes out of paper or card. Both the x and y axes should span from -5 to 5. Players arrange their papers such that one is standing vertically against a stack of books and the other is resting in front of it on a flat surface, hidden from the view of the other player.

Players secretly place their ships on the flat coordinate plane in front of them. Ships are placed across several coordinate points, but may not be placed on a diagonal. For example, Player 1's battleship is placed horizontally across 4 points, from (-2,-3) to (2,-3). Each player has 5 ships of lengths 2, 3, 3, 4 and 5. Ships should be clearly marked using bottle caps, colored pen or some other indicator and cannot be moved once placed.

Player 1 begins by calling out a coordinate point. If the point lies on one of Player 2's ships, he or she says, "Hit," otherwise it is a "Miss." Player 1 uses his or her vertical coordinate plane to keep track of hits and misses on Player 2's ships. Players go back and forth until all of one player's ships are sunk.

Extensions:

- Apply to geometry by using shapes instead of ships. Players must hit every corner in order to sink the shape.
- Apply to relations and functions. Instead of placing ships, students graph a line on their coordinate plane, such as $y = x + 3$. All points on the graph must be hit, and students must state the relation or function of their opponent in order to win. How many points must be hit in order to find the equation of the line?

5.1.6 20 Questions

Applicable Topics: Numbers (I), Fractions, Decimals and Percentages
Number of Players: 2

How to Play:
Player 1 thinks of a number and gives a range of possibilities to Player 2, e.g. "I'm thinking of a number between..." 0 and 100, -20 and -10, 1,000 and 2,000, etc. Player 2 must ask questions to try and guess the number. Player 1 may only answer "yes" and "no." Player 2 must ask questions like:

- "Is it larger than 50?"
- "Is it smaller than 10?"

Keep count of how many questions it takes to guess the number. Every question counts as one point. Each player gets a few rounds to choose a number and a few rounds to be the guesser. The player with the lowest score wins.

Extensions:

- Play variants of the game where students must choose a fraction, decimal or percentage.

5.1.7 Guess Who?

Applicable Topics: Numbers (I), Fractions, Decimals and Percentages
Number of Players: 2

How to Play:
Players begin with a 10×10 card on which are written the numbers 1-100. Each player secretly chooses a number and writes it on a small paper to keep in front of them during the game. Be sure the other player doesn't see your number!

Players take turns asking each other questions regarding properties of the other player's number. Questions can only be answered by "yes" and "no." Examples of questions may include:

- "Is your number a prime number?"
- "Is your number a square number?"
- "Is your number an odd number?"
- "Is your number a multiple of 3?"
- "Is your number a factor of 10?"

Students cross of numbers with an X or cover up numbers with markers as they eliminate possibilities from each other's answers. Play until someone wins best of 5.

Extensions:

- Make special game cards for a game on fractions, decimals or percentages, as long as both players' cards contain the same numbers.

5.1.8 Snap!

Applicable Topics: Congruence, Similarity, Fractions, Decimals and Percentages.
Number of Players: 2-5

How to Play:
You will need to make a pack of at least 40 cards. On each card write a fraction, a decimal or a percentage. Make sure there are several cards which carry equivalent fractions, decimals or percentages.

Shuffle the cards and deal them out, face down to the players. Place a baton-like item, such as a pen, ruler or spoon, at the center of the table so that it is within equal reach of all of the players. Players go in a circle, placing one of their cards face up in the middle. The first player to see that a card is

equivalent to another card face up in the middle must grab the baton and then, if correct, wins all of the cards in the middle. If, however, a player grabs the baton and there are no equivalent numbers showing, he or she must give one card to each remaining player. Players are eliminated when they run out of cards, but may still "grab in" to get back in the game if they can identify equivalent numbers played by the remaining players. The game continues until one player has won all of the cards.

Extensions:

- Apply to topics like Congruence and Similarity by using cards with shapes drawn on them and having players identify congruent or similar shapes.
- As an alternative to grabbing a baton, have students shout, "Snap!" when they identify matching cards to minimize injuries and chaos.

5.2 Stimulating Puzzles

5.2.1 SUDOKU

The classic Sunday paper SUDOKU puzzles are a wonderful tool for teaching logical thinking and problem-solving strategies to students. Each row, column, diagonal, and mini box must contain the numbers 1-9.

Do an easy example with students, and then give them more to do on their own. Stress the fact that there is only **one** possible number for each box - just because a number *could* fit does not necessarily mean that it *does* fit. Teach concepts like process of elimination to remove possibilities. Even if kids do not like math, these puzzles will help to make them better thinkers!

5.2.2 Magic Squares

A magic square is a 3 x 3 square of numbers in which every row, column and diagonal add up to the same total or "magic number." Here is an example of a Magic Square with 24 as its magic number:

11	3	10
7	8	9
6	13	5

Create your own magic square using the numbers 1-9 and a magic number of 15. How many ways can you do it?

There are 880 different solutions to a 4 x 4 magic square using the numbers 1-16. How many of them can you find where the magic number is 34?

Find the magic number in these Magic Squares and then complete them:

6		
7	5	3

		10
	7	
4		5

Now try these, a little harder, where numbers are given but the reasoning is not so straightforward:

14	3	
		13
8	15	

11	1	
9		7
	15	5

Solutions:

6	1	8
7	5	3
2	9	4

9	2	10
8	7	6
4	12	5

14	3	10
5	9	13
8	15	4

11	1	12
9	8	7
4	15	5

5.2.3 24 Squares

Surround a box by placing a digit on each side. Students must use all four numbers, together with any mathematical symbols they know, to produce an answer of 24. For example:

$$\begin{array}{ccc} & 3 & \\ 4 & \square & 5 \\ & 1 & \end{array} \qquad \begin{array}{ccc} & 5 & \\ 5 & \square & 5 \\ & 5 & \end{array} \qquad \begin{array}{ccc} & 8 & \\ 1 & \square & 9 \\ & 3 & \end{array}$$

Use this game to teach BODMAS as order of operations must be known to solve many of these puzzles. Arrange students in groups and see who can finish 10 the fastest.

Possible Solutions:

$$5 \times 4 + 3 + 1 = 24$$
$$5 \times 5 - (5 \div 5) = 24$$
$$8 \times (9 \div 3) \times 1 = 24$$

5.2.4 Four 4's

This easy to learn activity can provide many hours of interest and fun in your students. The idea is to express numbers from 1 to 100 using exactly four 4s and whatever mathematical symbols one knows. For example:

$$15 = 44 \div 4 + 4$$

or,

$$15 = (4 \times 4) - 4 \div 4$$

There are 15 ways to create 9. Teach BODMAS since order of operations is key to the answers. It also drills fractions and other mathematical concepts. Can anyone make it all the way to 100?

5.2.5 Make a Century

By putting arithmetical signs in suitable places between the digits, make the following statement correct:

$$1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9 = 100$$

There is more than one solution. See how many you can find.
Useful Tip: *Here are a few solutions:*

$$123 - 4 - 5 - 6 - 7 + 8 - 9 = 100$$
$$123 - 45 - 67 + 89 = 100$$
$$(1 \times 2 \times 3) - (4 \times 5) + (6 \times 7) + (8 \times 9) = 100$$
$$(1 \times (2 + 3) \times 4 \times 5) + 6 - 7 - 8 + 9 = 100$$

5.2.6 Intriguing Multiplication

Playing with his calculator one day, Jonny multiplies together the numbers 159 and 18 and obtains 7632. Upon reflection, he realizes that the equation

$$159 \times 48 = 7632$$

contains each of the digits from 1 to 9, only once each. He can hardly believe his luck and feels the results must be unique. But he is wrong! There are several other pairs of numbers whose product and result are such that all the digits are used only once. Can you find any of them?
Useful Tip: *Here are some more examples:*

$$138 \times 42 = 5796$$
$$198 \times 27 = 5346$$
$$186 \times 39 = 7254$$
$$157 \times 28 = 4396$$
$$1963 \times 4 = 7852$$

5.2.7 Fraction Factory

The Babylonians had no notation for fractions such as $\frac{2}{3}$ or $\frac{3}{5}$, but only for unit fractions (fractions with 1 on the top, such as $\frac{1}{2}$ or $\frac{1}{5}$). This meant that a fraction like $\frac{2}{3}$ would have to be expressed as a sum or difference of unit fractions:

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$$

Can you find ways of expressing fractions as the sum of difference of different unit fractions?
Here are some examples:

$$\frac{1}{12} = \frac{1}{3} - \frac{1}{4}$$

$$\frac{2}{5} = \frac{1}{5} + \frac{1}{6} + \frac{1}{30}$$

$$\frac{3}{4} = \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{20} + \frac{1}{24} + \frac{1}{30} + \frac{1}{120}$$

Useful Tip: *One approach is to just add and subtract different unit fractions to see what results. But to make real progress, certain patterns need to be recognized. For example:*

$$\frac{1}{12} = \frac{1}{3} - \frac{1}{4}$$

is a special case of

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

5.3 Brain Teasers and Mind Benders

5.3.1 Hundreds, Tens and Units

Take any three digit number such as 235. Write down the number formed by putting its digits in reverse order: 532. Subtract the smaller number from the larger number.

$$532 - 235 = 297$$

Now write down the number formed by reversing the order of the digits in the answer and add to the answer itself.

$$297 + 792 = 1089$$

When you have tried this for a few more numbers you will be able to predict the number and baffle friends.

Answer: *The answer is always 1089 unless the first number chosen has equal hundreds and ones digits, in which case the first subtraction would yield zero.*

5.3.2 Roll a Penny (or 100 /= Coin)

Penny A is rolled around Penny B without slipping until it returns to its starting point. How many revolutions does Penny A make?

Answer: *A makes 2 revolutions. Starting on the left side of Penny B, A will be upside-down when it has rolled to the top of B, upright when it is on the right side of B, upside-down again when at the bottom of B, and upright again when back at the start.*

5.3.3 Invert the Triangle

A triangle of bottle caps is made with one cap on the top row, 2 on the second row, 3 on the third and 4 on bottom. What is the smallest number of caps that must be moved to turn the triangle upside-down?

Answer: *Three caps need to be moved. Move the three corners: the top one to the right of the row below it, the bottom-left one up so now there are 4 caps on top, and the bottom-right moves to the very bottom. The row of three was left unchanged, and the original row of 4 now has 2 caps.*

5.3.4 Dividing Lines

Investigate the greatest number of regions you can make on a sheet of paper using a given number of lines. Create a table to track your results:

Number of Lines	1	2	3	4	5	6	7
Number of Regions							

Have students complete the table above and compare their results. Who was able to make the most regions? (*Hint: One can make 7 regions with 3 lines.*

Answer:

Number of Lines	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Number of Regions	**2**	**4**	**7**	**11**	**16**	**22**	**29**

5.3.5 The Ingenious Milkman

A milkman has only a 5 liter jug and a 3 liter jug to measure out milk for his customers. How can he measure 1 liter without wasting any milk?

Answer: *First fill the 3 liter jug. Next pour the 3 liters from this jug into the 5 liter jug. Again fill the 3 liter jug and then pour from it into the partially filled 5 liter jug until it is full. This leaves exactly 1 liter in the 3 liter jug. It would be possible to measure any whole number of liters by measuring single liters in this way. Clearly there are more efficient ways of measuring most quantities (5 and 3 can be done directly, 6 = 3 + 3, 8 = 5 + 3, etc. How about 7 liters or 4 liters?*

5.3.6 A Weighing Problem

A grocer has a pair of scale pans and 4 weights. The weights are such that with them she can correctly weigh any whole number of kilograms from 1 to 40. How heavy is each weight and how can he manage to weigh all of the different weights?

Answer: *The weights are 1 kg, 3 kg, 9 kg and 27 kg. By putting the weights on either scale pan, all of the weights from 1 to 40 can then be achieved. For example:*

$$11 = 9 + 3 - 1$$
$$20 = 27 + 3 - 9 - 1$$

5.3.7 A Question of Balance

In a box there are 27 new red ping pong balls, all looking exactly alike. However, it is known that one of them is faulty and weighs more than the others. Given that you have a balance but no weights, show how, by comparing sets of balls against each other, you can find the faulty ball in only 3 balances.

Answer: *Compare 9 balls with 9 balls and leave 9 in the box. If the scales balance then the heavy ball is in the box, but if not, then the 9 balls which weigh down the scale contain the heavy ball. Either way, the faulty ball has been narrowed down to a set of 9 after the first balance. Divide this set of 9 into three sets 3. After this second balance you will have narrowed the faulty ball down to a set of 3, and one more balance identifies the faulty ball.*

A similar but much harder problem is to find the odd ball from a set of 13 in only three balances.

Hands-On Tools from Locally Available Materials

Mathematics is a hands-on subject. Students must touch one-half, straight lines, negative numbers, circles, squares, cylinders, decimals, numbered items and shapes, and all other concepts in order to learn mathematics properly. There is a limitless number of items that can be made to teach mathematics. These items can be made from local materials, frequently from left-over items so that there is no additional cost in money.

Each tool is explained in terms of how to make and then how to use. If science needs local materials and student-centered learning to discover, then math, as the core of science, must be taught in the same manner. Every science syllabus uses mathematics!

6.1 Accounts

How to Make: Use large manila posters to make T-accounts tables. Make removable paper strips with tape to place transactions under the debit (Dr) or credit (Cr) sides, and account headings (Capital, Sales, Purchases, etc.) for opening new accounts.

Uses:

- Students physically place and move items to show cash flow among the different accounts.
- Use a paper or toy bus, the Transaction Express, to demonstrate the double entry rule of bookkeeping.
- See more in Topics and Activities for the Form III topic of Accounts.

6.2 Cartesian Plane

How to Make: Use an empty seed bag (mfuko), a sheet of manila paper or a wire grid to draw a large Cartesian plane with x and y axes. Points can be little dots or bottle caps with tape on the back, which can easily be moved around. Then connect with string for lines, etc.

Uses:

- Create Dice that can be used to plot random points. Make four dice out of manila paper (2 red - one of +'s and -'s and one of the numbers 1 - 6, and then 2 blue in the same way). Students roll the dice to find an x-coordinate (blue) and a y-coordinate (red) and then must plot the point on the Cartesian plane. Use this method to generate coordinate points for problems on slope, equation of a line, midpoint, distance formula, etc.
- Students are challenged by plotting $(1, 0)$; $(0, 1)$ and many such points, so practice and drilling are essential before trying to teach functions or relations.
- One can find the midpoint or any other properties of a line segment. Leaving a Cartesian plane in the classroom, students can use it during private studies. The tangible Cartesian plane is so essential because a 2-dimensional plane needs tangible interaction for learning.
- A reusable Cartesian plane saves much time in the classroom compared to drawing it on the board each time you teach a concept. Accuracy is so essential in teaching midpoint and related topics.

6.3 Circles

How to Make: Use bike wheels or bucket lids to draw circles on the board. Have students use coins or toilet paper rolls to draw them in their notebooks.

Uses:

- Wrap string around the wheel or lid to show circumference. Use another string to measure diameter, and use the ratio of the lengths to derive π. Compare results from different groups in class.
- Students can use a wheel to find the perimeter around the school or their home.
- See more in Topics and Activities for the Form III topic of Circles.

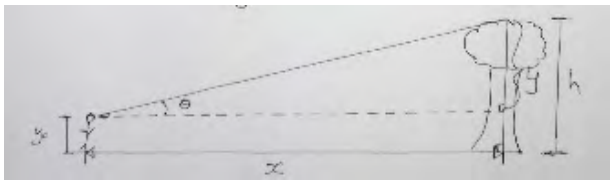
6.4 Clinometer

How to Make: Gather a protractor, a hollow pen tube or straw, a short piece of string (15 cm), some tape and a nail or some other small weight.

Tie the weight to one end of the string and tie the other end around the center of the pen tube. Tape the pen tube along the straight edge of the protractor so that the string hangs vertically downward through the center (make sure it is secured in place).

Uses:

- *Trigonometry*: The clinometer can be used to find the height of a tree or tall building. A student stands some distance away from the tree or building and looks through the pen tube so that he/she can see the highest point of the object. Another student reads the angle made by the string with the vertical 90° line on the protractor. (Note that, for an angle of elevation θ, the angle read from the protractor will actually be $(90° - \theta)$, since the string bob starts vertically downward at 90° rather than 0°.) Knowing the horizontal distance from the student to the object, as well as the height of the student by using a tape measure, one can use the properties of trigonometry to calculate the height of the object!



$$\tan\theta = \frac{y}{x}$$
$$y = x\tan\theta$$
$$h = y_o + y$$

Help students to prepare a data table as shown below, and then split them up into groups to go collect and record their data and calculate the height of a building. Have them compare their answers and identify sources of error. If possible, obtain the actual height and see how the calculated values compare. This is a great Math Practical for Form IV students! Which professions might use this method of measurement?

Measurements			**Calculations**			
x (m)	y_o (m)	$(90° - \theta)$ (°)	θ (°)	$\tan\theta$	$y = x\tan\theta$ (m)	$h = y_o + y$ (m)

6.5 Compass

How to Make: Use a strip of folded paper with holes at various lengths along the paper. Keep one pen secure at one end and use another pen or pencil to trace a circle. Alternatively, use a length of string tied around two pens at either end. Unroll the connecting string to the desired length and secure one pen to be the center of the circle and use the other to trace the circumference.

Uses:

- Have each student make one of these paper compasses for taking notes, especially during topics when they need to draw many, many circles. Otherwise they will be sharing coins or compasses and it will take a long time for them to copy your notes.

6.6 Dice

How to Make: Take an empty chalk container and write numbers on the sides. Or you can make a box using manila paper, or even make cubes out of bars of soap. Have your students help to make them - they love helping with these projects.

Uses:

- *Probability*: Students love to play and see probability happen rather than just memorizing formulas. Chart the results of using dice. Then discover the formulas of probability.
- *Coordinate Geometry*: Use dice to generate random numbers for plotting points, graphing lines, etc.
- *Numbers*: Use random numbers to drill students on multiplication, addition, fractions, decimals, etc.

6.7 Dominoes

How to Make: Use a little rectangular paper marked in 2 sections. For example, one section has a number four and the other section has a number six.

Uses:

- From the example, the ordered pair $(4, 6)$ can be written. This random point can be graphed on a Cartesian Plane.
- These domino cards can also be used for random pairing of numbers as they learn multiplication tables. These must be drilled in many ways, because older students often still use tables on their exercise books, which will not be available to them during exams.

6.8 Flash Cards

How to Make: Small cards or paper rectangles can be used.

Uses:

- Students enjoy playing Around the World in the classroom as multiplication tables are drilled.
- These cards can be used for board races with 6 or more students at the board responding to the flash card. All the students at their desks can also be responding to the question.
- Use flashcards with pictures or math terms written on them to quiz students on new terminology.

6.9 Fractions and Decimals

How to Make: Empty water bottles are excellent for teaching fractions and are very meaningful. Each student can supply an empty bottle from home to use during class.

Sticks can be put to better use in math class than for discipline. Have the students collect them and they will become fractions. Now the students have an object of one-half, one-third or any fractional component. After students understand these fraction concepts, there are activities on Fractions which may be used.

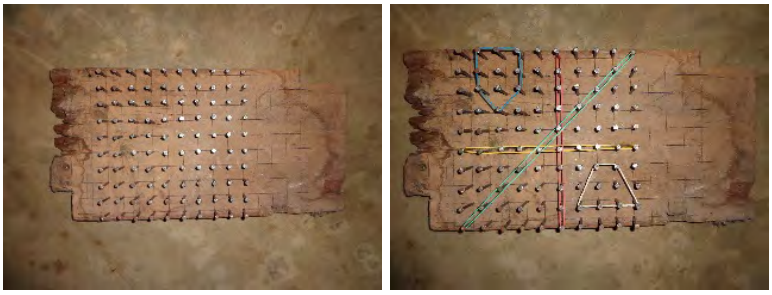
Uses:

- Have students fill up half a bottle, one-third of a bottle, etc. Students can continue to add one-half and one-half to get a full bottle as they work together in pairs with their bottles of water.
- Students can then subtract with fractions in the same way. What learning! Have them write all the fraction addition and subtraction problems they have solved. Creating fraction equations with the sticks and then solving them will help the students understand the operations on fractions.
- Relating these fractions to cooking will be very practical and meaningful. To cook, one measures one cup, a half cup, a third cup, etc. What happens when we must cook for twice the number of people?
- Convert these learning exercises to decimals. Students need to see and touch 0.5 or 0.3 or 0.25 to learn the meaning of it. Sometimes even money can be used.

6.10 Geoboard

How to Make: Gather nails and left-over square slabs of wood of various sizes from a local carpenter, or have students supply these items from their homes. Place nails in a grid at one inch or 2 centimeters apart from each other, across and down to get a square. The number of nails used will be based on the size of the piece of wood. A size of 10 by 10 is excellent.

Many times a carpenter will share these items to increase the learning experience of students, or your school may share. The carpenter knows how often numbers are used in his profession! Every student can build a geoboard, or one geoboard for a group of 4 students. It is good to involve the parents in a day school.



Uses:

- *Perimeter*: The formula for perimeter around any size rectangle can be demonstrated. For example, for a rectangle of 5 inches by 4 inches, one can use rubber bands to mark off the given dimensions, and then one can count 18 units of perimeter to see that:

$$2 \times \text{Length} + 2 \times \text{Width} = \text{Perimeter}$$

- *Area*: The formula for area can be demonstrated by marking a rectangle of 5 inches by 4 inches or any dimension. Use rubber bands to mark the square. Count the square units in the surface of the rectangle - it will be 20 square units. This will allow students to discover the formula:

$$\text{Length} \times \text{Width} = \text{Area}$$

- *Coordinate Geometry*: Lines can be graphed on a Cartesian Plane. First mark the x-axis and y-axis. It is good to use different colors of rubber bands (yellow for y-axis and red for x-axis). Use little dots of paper to plot the points. After plotting 3 or 4 points, use a rubber band to connect them. The students enjoy such hands on activities.

  After a line is plotted, one can demonstrate slope, or gradient. Slope is the change of y over change of x and can be counted and demonstrated on a geoboard.

  Parallel lines can be placed on a geoboard with rubber bands. Then one can use the points to see that the slopes are equal for these two lines. Also try it with perpendicular lines.

- *Relations and Functions*: After tables of values are created, students can plot them on the geoboard. Begin by graphing the line using two points from the table. Then one can find the domain and range. The inverse can be discovered and discussed as well. All of this on relations can also be applied to functions.

6.11 Globe

How to Make: Have students make spheres by binding thin strips of bamboo together with string. Make sure they have axes through the center and a few outer rings. You can fill in the empty sections with paper to illustrate various angles within the sphere.

Uses:

- *Earth as a Sphere*: Use the spheres in class to teach students about longitude and latitude, great circles, the equator and prime meridian, and various angles measured within the sphere. Color-code different properties and have students try to label them.

6.12 Number Line

How to Make: Create a number line in a variety of ways. The best may be a piece of string or ruler. Then have numbers written on little dots that can be placed along the number line in positive and negative directions.

Students can be a number line in class. Thy can hold a positive or negative number. Involvement is the best way to learn.

Number lines can be drawn outside in the dirt for even more interactive learning.

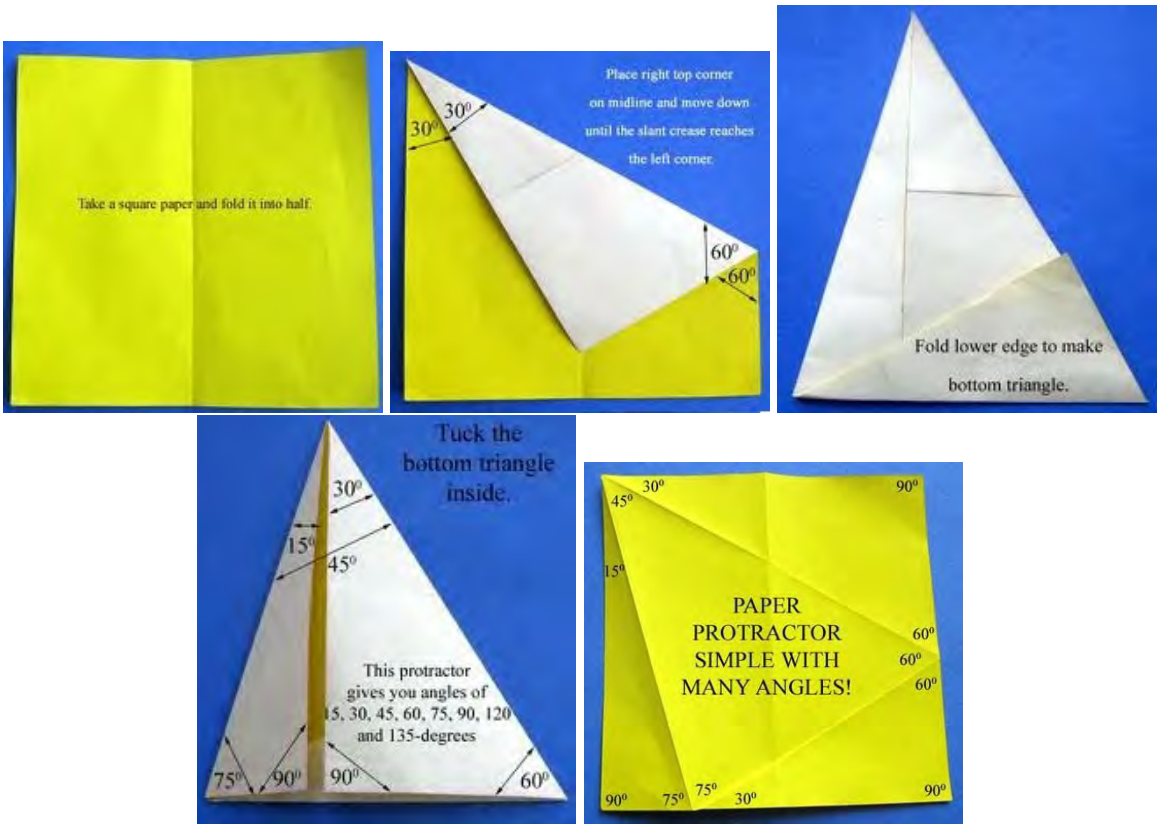
Uses:

- Use the "hopping method" of movement as students process $-3 + 1 = -2$. Have one student at a time stand in front of the student number line and hop to the left and right to add and subtract integers. It is good to create a hopping song while learning these negative integers for addition and subtraction.

- Inequalities are graphed on a Number Line. These graphs are the stepping stone to graphing on an x-y plane. It also demonstrates which numbers are larger or smaller. Students need to see to understand such concepts of greater than or less than.

6.13 Protractor

How to Make: Take a square piece of paper and fold it in half into two vertical rectangles (picture 1). Take the top right corner and fold it down to the center line (picture 2). Next, bring the bottom left corner to the same point on the center line so that the left edge meets along the edge just folded (not pictured). Then fold up the lower edge to make the bottom triangle (picture 3). Finally tuck the bottom triangle inside (picture 4). Leave the triangle folded like this, or unfold (picture 5) to view the angles created. Label the angles as shown - this protractor produces angles of 15°, 30°, 45°, 60°, 75°, 90°, 120° and 135°.



Uses:

- *Geometry*: Students can use the paper protractor to construct various acute, obtuse, right and reflex angles.

6.14 Spinners

How to Make: Gather a sheet of paper or cardboard, a pen/pencil, paper clip, Protractor and Compass.

Draw a circle on the paper using the compass. Use the protractor to divide the circle into a desired number of equal-sized sections (have students determine the angle necessary for each section based on the number of sections desired). Label each section with a number, color or some other identifier. Hold the pen/pencil in the center of the circle with the paper clip free to spin between it and the paper. Flick the paper clip to start the spinner and see where it lands.

Uses:

- *Probability*: Create a spinner with different-sized sections and have students determine the probability that it will land on a given one. Conduct several trials to lead students to discover formulas for probability.
- *Fractions and Decimals*: Label each section with a different fraction or decimal. Or divide the spinner into unequal sections and label each one according to its fraction of the whole circle. Have students perform operations on fractions and decimals that the spinner lands on.

6.15 Three-Dimensional Objects

How to Make: Use paper or a wire grid to construct cylinders, prisms and pyramids. Use empty cardboard boxes or chalk boxes to demonstrate interior angles formed by various planes. Use old Nido and oatmeal tins as cylinders.

Uses:

- Students get great enjoyment out of constructing these three-dimensional objects themselves. Use them to derive formulas on surface area and volume.
- Tape a sheet of paper around a large cylindrical tin and use circular lids to show how to derive the surface area of a cylinder.
- See more in Topics and Activities for the Form IV topic of Three-Dimensional Figures.

Glossary of Math Terms in Kiswahili

Section	English	Kiswahili	Example
Numbers & Operations	approximately	kadiri ya	
	average	wastani	
	bar graph	grafu kwa mihimidi	
	decimals	desimali	sifuri nukta moja (0.1)
	denominator	asili	
	equal to	sawasawa na	x sawasawa na 4
	even	shufwa	namba shufwa
	factor(s)	kigawo (vigawo)	Vigawo vya 4 ni 1, 2, na 4.
	fraction(s)	sehemu	
	greater than	kubwa kuliko	
	hundreds	mamia	
	inverse	kinyume	kinyume cha 12 katika kuzidisha ni 1/12
	less than	mdogo kuliko	
	multiple(s)	kigawe (vigawe)	
	negative	hasi	
	numeral, digit	tarakimu	
	numerator	kiasi	
	odd	witiri	namba witiri
	ones	mamoja	
	parentheses, bracket(s)	bano (mibano)	
	percent	asilimia	asimilia tano (5 %)
	place value	kiwango cha tarakimu	
	positive	chanya	
	power(s)	kipeo (vipeo)	1 kipeo cha pili
	prime	tasa	namba tasa
	product(s)	zao (mazao)	
	ratio	uwiano	
	remainder	baki	kumi baki moja
	roman numerals	namba za kirumi	
	root(s)	kipeuo (vipeuo)	
	square	namba mraba	16 ni namba mraba ya 4
	square root	kipeuo cha pili	kipeuo cha pili cha 9
	statistics	takwimu	
	ten thousands	makumi elfu	
	tens	makumi	
	thousands	maelfu	
	to add	kujumlisha	
	to count	kuhesabu	
	to divide	kugawanya	
	to multiply	kuzidisha	
	to subtract	kutoa	
	total	idadi	
	whole number	namba nzima	
Algebra/ Aljebra	axis	jira (majira)	
	coefficient(s)	kizigeu (vizigeu)	
	equation	mlinganyo	

Section	English	Kiswahili	Example
	expression	uonyesho	uonyesho wa kialjebra
	function	kazi	
	like terms	mitajo inayofanana	
	term(s)	mtajo (mitajo)	
	to substitute	kuingiza	
	unlike terms	mitajo isiyofanana	
	value	thamani	ikiwa $m = 2$, $n = 3$, tafuta thamani ya mn^2
	variable(s)	kigeu (vigeu)	
Geometry/ Jometri	acute	pembekali	
	angle	pembe	
	area(s)	eneo (maeneo)	
	base(s)	kitako (vitako)	
	circle	duara	
	cone	pia	
	cylinder	mche duara	
	degrees	nyuzi	pembemraba ina nyuzi 90
	diameter	kipenyo	
	diamond	tiara	
	direction	mwelekeo	
	equilateral triangle	pembetatu sawa	
	height(s)	kimo (vimo)	
	horizontal	ya mlalo	
	isosceles triangle	pembetatu pacha	
	length	urefu	
	line(s)	mstari (mistari)	
	lines of symmetry	mistari ya pacha	
	magnitude	uzito	
	obtuse	pembebutu	
	origin	peo	
	oval	duara dufu	
	parallel	sambamba	mistari msambamba
	parallelogram	msambamba	
	perimeter, circumference	mzingo	
	perpendicular	wima	mstari wima
	piece	kipande	
	point	nukta	nukta A hadi nukta B
	protractor	kipimapembe	
	pyramid	mche pembetatu	
	radius	nusu kipenyo	
	rectangle	mstatili	
	right	pembemraba	
	segment	sehemu	
	shape(s)	umbo (maumbo)	
	side	upande	
	sphere	tufe	
	square	mraba	
	straight	nyoofu	
	surface	bapa	
	to connect (points)	kuunganisha	
	to reflect	kuakisi	
	to rotate	kuzungusha	
	trapezoid	trapeza	
	triangle	pembetatu	

Section	English	Kiswahili	Example
	vector(s)	mwale (miale)	
	vertical	ya wima	
	volume	ujazo	
	width	upana	upana wa ...
Others	always	siku zote	
	chart, table	chati	
	chart, table	jedwali	
	diagram(s)	mchoro (michoro)	
	diagram(s)	kielelezo (vielelezo)	
	difference	tofauti	
	example	mfano	kwa mfano ...
	gram, kilogram, ton	gramu, kilogramu, tani	
	group	kukusanya	
	in total	kwa jumla	
	increment	ongezeko, nyongeza	
	interest	riba	
	list	orodha	
	liter, mililiter	lita, mililita	
	loss	hasara	
	measurement(s)	kipimo (vipimo)	vipimo vya metriki
	meter, centimeter, kilometer	meta, sentameta, kilometa	
	never	abadan	
	note	tarbihi	Tarbihi: ...
	often	mara nyingi	
	principle, rule, law	kanuni	
	profit	faida	
	ratio	uwiano	
	relationship	uhusiano	uhusiano wa vipimo vya metriki
	series, succession	mfululizo	
	series, succession	kwa mpangilio	
	sometimes	pengine	
	step(s)	hatua	
	symbol	alama	
	to approximate	kukadiria	
	to arrange	kupanga	kupanga kuanzia ndogo hadi kubwa zaidi
	to be equal, symmetrical	kulingana	
	to change, convert	kubadili	Badili sehemu kuwa desimali.
	to compare	kulinganisha	
	to describe	kueleza	
	to draw	kuchora	
	to end at	kuishia	
	to estimate, guess	kisia	
	to find	kutafuta	Tafuta x
	to gather	kukusanya	kukusanya data
	to guess	kukisia	
	to list	kuorodhesha	
	to remain	kubaki	
	to represent	kuwakilisha	
	to show	kuonyesha	
	to simplify	kurahihisha	
	to start at	kuanzia	

Section	English	Kiswahili	Example
	unit(s)	kiambisho (viambisho)	
	use(s), application(s)	ma(tumizi)	
	usually	aghalabu	
	way, method, solution	njia	njia hii, njia ndefu, njia fupi, njia nyingine
	weight	uzani	