UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

042

ADDITIONAL MATHEMATICS
(For Both School and Private Candidates)

TIME: 3 Hours

2006/10/23 p.m.

Instructions

1. This paper consists of sections A and B.
2. Answer **all** questions in section A and **four (4)** questions from section B.
3. Mathematical tables and formulae may be used.
4. Electronic calculators are **not** allowed in the examination room.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet (s).

SECTION A (60 marks)

Answer **all** questions in this section

All working for each question must be shown clearly

1. (a) Simplify using basic properties of sets:

(i) $(A' \cap B') \cup (A' \cap B)$ **(1 ½ marks)**

(ii) $(A - B) - \cap (B \quad A)$ **(1 ½ marks)**

(b) In a class of 28 pupils, 5 study both chemistry and physics, 7 study both chemistry and biology and none study both physics and biology. The number of pupils who study physics is equal to the number of those who study biology. 16 pupils study chemistry and 2 study none of the three subjects. Find the number of pupils who study

(i) biology only

(ii) chemistry only **(3 marks)**

2. (a) If p is the statement "$x = 1$" and q the statement "$x^2 = 1$". Write the following statements in symbolic form:

(i) $x^2 \neq 1$ because $x \neq 1$.

(ii) $x = 1$ is a necessary and sufficient condition for $x^2 = 1$.

(iii) If $x^2 \neq 1$ then $x \neq 1$. **(1 ½ marks)**

(b) By using truth tables show that

$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$ **(4 ½ marks)**

3. (a) (i) Calculate the inverse M^{-1} of the matrix $M = \begin{bmatrix} x & x-1 \\ y & y \end{bmatrix}$ where $y \neq 0$.

(ii) Find the values of x and y such that $M^{-1}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \end{bmatrix}$ **(3 marks)**

(b) (i) Find the value of k for which the simultaneous equation

$2x - ky = 2$

$3x + (k+1)y = 4$

has no solution.

(ii) By using Cramer's rule find the values of x and y in (b (i)) above when $k = -3$.

(3 marks)

4. The cumulative frequency table for scores of 55 pupils in a particular test is give below:

Not more than	10	20	30	40	50	60	70	80	90
Cum. frequency	3	8	17	X	33	41	46	49	53

If the two classes bounded by the scores of 40 have equal number of frequencies

(a) find x **(1 mark)**

(b) prepare the corresponding class frequency table and calculate the mean of the distribution **(03 marks)**

(c) determine the modal classes of the distribution. **(2 marks)**

5. (a) If $f(x) = \frac{x-1}{x+1}$, show that $f\left[\frac{1}{x}\right] = -f(x)$ **(1 mark)**

(b) Find the domain and range of the functions

(i) $g(x) = \frac{x}{2 + x^2}$ **(2 ½ marks)**

(ii) $h(x) = \frac{1}{\sqrt{x-4}}$ **(2 ½ marks)**

6. (a) prove for any two angles A and B that

$$(\cos A + \cos B)^2 + (\sin A + \sin B)^2 = 2[1 + \cos(A - B)] \quad \textbf{(2 marks)}$$

(b) Solve for θ between 0° and 180° given that $6\cos^2\theta - \cos\theta - 1 = 0$. **(2 marks)**

(c) If $A + B = \frac{2}{3}\pi$ radians, and $A - B = \frac{1}{3}\pi$ radians evaluate

$(\sin A + \sin B) + \sqrt{3}(\cos A + \cos B)$ **(2 marks)**

7. (a) The rate of working, P watts of an engine which is travelling at a speed $V ms^{-1}$ is given by

$$P = 10V + \frac{4000}{V}, \quad V > 0.$$

Find the speed at which the rate of working is least. **(3 marks)**

(b) If $y = \sqrt{1+\sin x}$, show that $\frac{dy}{dx} = \frac{1}{2}\sqrt{1-\sin x}$ **(3 marks)**

8. (a) (i) Evaluate $\int_0^{\frac{\pi}{4}} \sin^2 2x \cos 2x \, dx$

(ii) Find $\int \frac{1-3x}{\sqrt{x}} \, dx$ **(3 marks)**

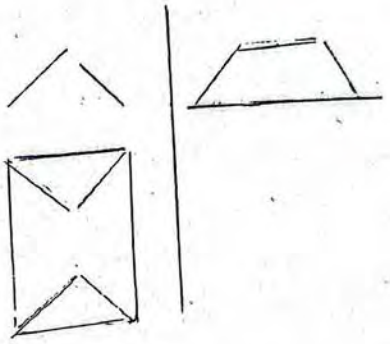
(b) The gradient of a curve at the point (x, y) on the curve is given by $(2x - 4)$.
If the minimum value of y is 3, find the equation of the curve. **(3 marks)**

9. (a) Find the diameter and the coordinates of the centre of the circle $x^2 + y^2 - 4x + 6y - 12 = 0$. (3 marks)

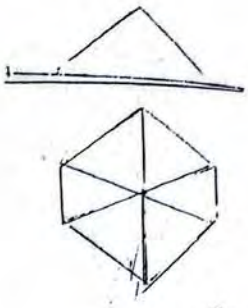
(b) Show that the line $4x - 3y - 5 = 0$ is a chord of the circle $x^2 + y^2 + 3x - y - 10 = 0$. Hence find its length. (3 marks)

10. (a) Name the objects whose plan and elevations are give below:

(i)

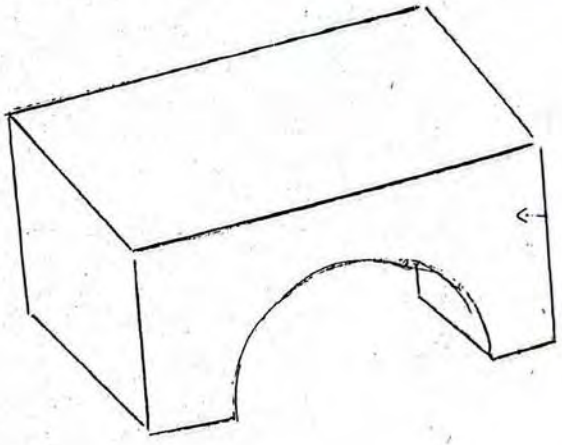


(ii)



(2 marks)

(b) Sketch the plan and elevation of the following sold from the direction mark "front"



(4 mark

SECTION B (40 Marks)

Answer **four** (4) questions from this section. All working must be shown clearly.

11. (a) A two digit numeral is written by choosing the tens digits from the set {1, 2, 3 and the unit digits from the set {5, 6}. What is the probability that in performi experiment the outcome will represent a number greater than 30? (4 marks

(b) In a field there are 30 sheep, 15 of which are white and 15 black. If 6 of the shee selected at random, what is the probability that three of them are white and three black? (6 marks

12. (a) Given the vectors $\underline{a} = 3\underline{i} + 2\underline{j} + 9\underline{k}$ and $\underline{b} = \underline{i} + P\underline{j} + 3\underline{k}$ find the value (4 marks

(i) $\underline{a}$ and $\underline{b}$ are parallel

(ii) $\underline{a}$ and $\underline{b}$ are perpendicular.

(b) Determine the unit vector perpendicular to both vectors $\underline{u} = 3\underline{i} + \underline{j} - 2\underline{k}$ and $\underline{v} = 2\underline{i} + 3\underline{j} - \underline{k}$ **(3 marks)**

(c) Given the vector $\overrightarrow{PQ} = 2\underline{i} + \underline{j}$ where P is the point (-3, 2), determine the coordinates of Q. Hence show that $\overrightarrow{OQ}$ is parallels to $\overrightarrow{OR}$ where R is the point (-2, 6) **(3 marks)**

13. (a) The square whose vertices are the point A(0,3) B(1, 1), C(3, 2), D(2, 4) is mapped to the plane figure A′, B′, C′ D′ by $M = \begin{bmatrix} 1 & -2 \\ 1 & 2 \end{bmatrix}$. Find the area of A′ B′ C′ D′ **(2 marks)**

(b) A transformation T assigns to any point (x, y) an image (x′, y′) according to the rule

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \end{bmatrix}$$

(i) Find the equation of the image lines obtained when all points on the lines $y = 2x$ and $y = x - 3$ undergo the transformation T **(5 marks)**

(ii) Prove that the line $y = 3x + 1$ maps onto itself under transformation T. **(3 marks)**

14. A retailer is about to stock his shop with TV sets. Set A costs 100,000/= each and occupies a volume of 1 unit. Set b costs 80,000/= and occupies a volume of 2 units. The total money available for purchasing stock is 4,000,000/=. The total room available is 70 volume units. If x of A and y of B are bought, find the system of inequalities which must be satisfied, if the profit on the sale of each set A is 10,000/= and of each set B is 15,000/=. Find the maximum profit that can be obtained when the shop has been correctly stocked and all the stock is sold. **(10 marks)**

15. (a) Box M contains 5 pieces of paper numbered 1, 3, 5, 7, 9. Box P contains 3 pieces of paper numbered 1, 4, 9. One piece of paper is removed at random from each box. Calculate the probability that the two numbers obtained have

(i) the same value **(4 marks)**

(ii) a product that is exactly divisible by 3.

(b) The events A and B are such that $P(A) = \frac{2}{5}$, $P(A/B) = \frac{3}{8}$ and $P(B/A) = \frac{1}{2}$

Find

(i) $P(A \cap B)$ **(4 ½ marks)**

(ii) $P(A' \cap B')$

(c) In how many ways can a committee of 5 people be chosen out of 9 people? **(1 ½ marks)**

16. (a) Find the coordinates of the turning points of the curve $y = (2x - 3)(x^2 - 6)$ and determine the nature of each point. **(4 marks)**

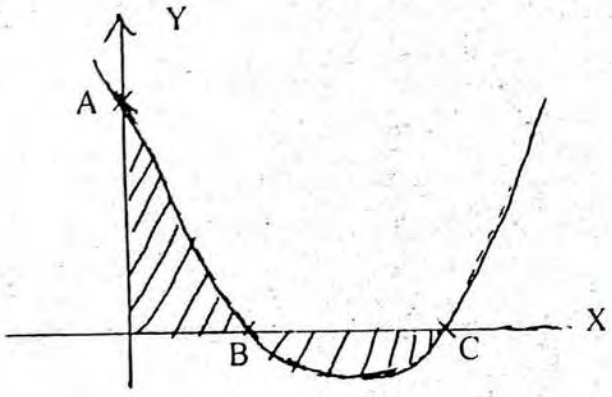
(b) A cylindrical container opened at one end, has height h cm and base radius r cm.

(i) Write the expression for total surface area of the container s cm^2 and the volume, $V cm^3$ in terms of h and r.

(ii) Given that s has the value 3π, from (i) above show that $V = \frac{1}{2}\pi r\left(3 - r^2\right)$.

(iii) Hence, find the value of r and the corresponding value of h which make V a maximum. **(3 marks)**

(c) The curve $y = x^2 - 6x + 5$ cuts the y – axis at A and the x – axis at B and C as shown in the figure below.



Find the coordinates of A, B and C and calculate the total area of the shaded parts. **(3 marks)**