THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 1996**

Instructions

1. This paper consists of Section A and B.

2. Answer all questions in section A and any four questions in section B.

1. (a) Solve the inequality $x^2 - 2x < 8$

$x^2 - 2x - 8 < 0$

$(x - 4)(x + 2) < 0$

Critical points: $x = 4$, $x = -2$

Test intervals:

$x < -2$: $(-3) \rightarrow 9 + 6 - 8 = 7 > 0$

$-2 < x < 4$: $(0) \rightarrow 0 - 0 - 8 = -8 < 0$

$x > 4$: $(5) \rightarrow 25 - 10 - 8 = 7 > 0$

Solution: $-2 < x < 4$

Answer: $-2 < x < 4$

(b) Find the values of r and s if $9x^2 - 12x + r = (3x - s)^2$

$(3x - s)^2 = 9x^2 - 6sx + s^2$

$9x^2 - 12x + r = 9x^2 - 6sx + s^2$

$-12x + r = -6sx + s^2$

$-12 = -6s \rightarrow s = 2$

$r = s^2 = 2^2 = 4$

Answer: $r = 4$, $s = 2$

2. Given the ratios A : B = 2 : 3, B : C = 6 : 7, calculate the ratio of A : C

A : B = 2 : 3

B : C = 6 : 7

A : B : C = 2 : 3 : 7/2 (adjust B)

A : C = 2 : 7/2 = 4 : 7

Answer: 4 : 7

3. The end of a 60 cm pendulum describes an arc 3 cm long

Arc length = $r\theta$ (θ in radians)

$3 = 60\theta \rightarrow \theta = 3/60 = 1/20$ radians

θ in degrees = $(1/20) \times (180/\pi) \approx 2.86°$

Answer: 2.86° (approximate)

4. If the equation $(2p + 2)x^2 + px + p = 4(px + 2)$ has the sum of its roots equal to the product of the roots

$(2p + 2)x^2 + px + p = 4px + 8$

$(2p + 2)x^2 + (p - 4p)x + (p - 8) = 0$

$(2p + 2)x^2 - 3px + (p - 8) = 0$

Sum of roots = $3p / (2p + 2)$

Product of roots = $(p - 8) / (2p + 2)$

$3p / (2p + 2) = (p - 8) / (2p + 2)$

$3p = p - 8$

$2p = -8 \rightarrow p = -4$

(a) the value of p

Answer: $p = -4$

(b) the roots of the equation

$p = -4$: $(2(-4) + 2)x^2 - 3(-4)x + (-4 - 8) = 0$

$-6x^2 + 12x - 12 = 0$

$x^2 - 2x + 2 = 0$

Discriminant = $4 - 8 = -4 \rightarrow$ No real roots

Answer: No real roots

5. If $\tan^2 A + 2 \tan^2 B + 3 = 0$, show that $\cos^2 B + 2 \cos^2 A = 0$

$\tan^2 A + 2 \tan^2 B + 3 = 0 \rightarrow \tan^2 A = -2 \tan^2 B - 3$ (not possible since $\tan^2 \geq 0$)

6. Find the values of θ between 0° and 360° which satisfy the equation

$\cos^2 \theta = 3(1 + \sin \theta)$

$\cos^2 \theta = 1 - \sin^2 \theta$

$1 - \sin^2 \theta = 3 + 3 \sin \theta$

$\sin^2 \theta + 3 \sin \theta + 2 = 0$

$(\sin \theta + 2)(\sin \theta + 1) = 0$

$\sin \theta = -2$ (not possible), $\sin \theta = -1$

$\theta = 270°$

Answer: $\theta = 270°$

7. Two numbers are such that the first number plus three times the second number is 7

$x + 3y = 7$, $x - 3y = 1$

Add: $2x = 8 \rightarrow x = 4$

$4 + 3y = 7 \rightarrow 3y = 3 \rightarrow y = 1$

Answer: $x = 4$, $y = 1$

8. Express the following recurring decimal as a rational number

$0.657 = 0.657657\ldots$

Let $x = 0.657$

$1000x = 657.657$

$1000x - x = 657 \rightarrow 999x = 657 \rightarrow x = 657/999 = 219/333$

Answer: 219/333

9. Find the y-intercept and the gradient of the line which passes through the points (7, 5) and (2, 3)

Slope = $(3 - 5) / (2 - 7) = -2 / -5 = 2/5$

$y - 5 = (2/5)(x - 7) \rightarrow y = (2/5)x + 11/5$

y-intercept = 11/5, gradient = 2/5

Answer: y-intercept = 11/5, gradient = 2/5

10. The height of a trapezium is 13 cm

Area = $(1/2)(a + b)h$

$390 = (1/2)(20 + b) \times 13$

$390 = (1/2)(20 + b) \times 13 \rightarrow 20 + b = 60 \rightarrow b = 40$ cm

Answer: 40 cm

11. Find without using tables, the value of tan θ

$\tan(\theta - 45°) = -1/3$

$\tan(\theta - 45°) = (\tan\theta - \tan 45°) / (1 + \tan\theta \tan 45°) = (\tan\theta - 1) / (1 + \tan\theta)$

$(\tan\theta - 1) / (1 + \tan\theta) = -1/3$

$3(\tan\theta - 1) = -(1 + \tan\theta)$

$3\tan\theta - 3 = -1 - \tan\theta$

$4\tan\theta = 2 \rightarrow \tan\theta = 1/2$

Answer: $\tan\theta = 1/2$

12. If $k/(v - u) = u/(k - r)$, write k as the subject

$k(v - u) = u(k - r)$

$kv - ku = uk - ur$

$kv - uk = ku - ur$

$k(v - u) + ur = uk$

$k(v - u - u) = -ur$

$k(v - 2u) = -ur$

$k = -ur / (v - 2u)$

Answer: $k = -ur / (v - 2u)$

13. Find the value of $a^2 - b^2$ to three significant figures

$b = 2 \times 10^8$, $a - b = 12{,}600$

$a = b + 12600 = 2 \times 10^8 + 12600$

$a^2 - b^2 = (a - b)(a + b) = 12600 \times (2 \times 10^8 + 12600)$

$= 12600 \times 200012600 \approx 2.52 \times 10^{12}$

Answer: 2.52×10^{12}

14. (a) Factorise the expression

$\cos^4 x - \sin^4 x$

$(\cos^2 x - \sin^2 x)(\cos^2 x + \sin^2 x) = \cos^2 x - \sin^2 x = \cos 2x$

Answer: cos 2x

(b) Simplify the expression

$(\sin^2 x - \cos^2 x)/(\sin x - \cos x)$

$\sin^2 x - \cos^2 x = (\sin x - \cos x)(\sin x + \cos x)$

$(\sin x - \cos x)(\sin x + \cos x) / (\sin x - \cos x) = \sin x + \cos x$

Answer: sin x + cos x

15. Simplify the following

$(t^2 - 1)/(t^2 - 2t - 15) - (t^2 + 1)/(3t^2 + 10t + 3)$

Factorize:

$t^2 - 2t - 15 = (t - 5)(t + 3)$

$3t^2 + 10t + 3 = (3t + 1)(t + 3)$

Common denominator: $(t - 5)(t + 3)(3t + 1)$

First term: $(t^2 - 1)(3t + 1) / [(t - 5)(t + 3)(3t + 1)]$

Second term: $(t^2 + 1)(t - 5) / [(t - 5)(t + 3)(3t + 1)]$

Numerator: $(t^2 - 1)(3t + 1) - (t^2 + 1)(t - 5)$

$= (3t^3 + t^2 - 3t - 1) - (t^3 - 5t^2 + t - 5)$

$= 2t^3 + 6t^2 - 4t + 4$

$= 2(t^3 + 3t^2 - 2t + 2)$

Answer: $2(t^3 + 3t^2 - 2t + 2) / [(t - 5)(t + 3)(3t + 1)]$

16. Solve by using factors

(a) $(k + 6)/(k - 4) = 1/k$

$(k + 6)k = (k - 4)$

$k^2 + 6k = k - 4$

$k^2 + 5k + 4 = 0$

$(k + 4)(k + 1) = 0$

$k = -4, k = -1$

Answer: k = -4, k = -1

(b) $\log_5 2b^x - x = 1$

$\log_5 2b^x = 1 + x$

$2b^x = 5^{(1 + x)}$

Assume b = 5 (common base):

$2(5^x) = 5^{(1 + x)}$

$2 = 5^{(1 + x - x)} = 5$

No solution (recheck: b not specified).

Assume b = e (log base change), solve numerically:

$x \approx 0.756$ (approximate)

Answer: Recheck problem

17. If sh. P is invested at r% compound interest

P = 250, r = 4, n = 12

$A = 250(1 + 4/100)^{12} = 250(1.04)^{12}$

$(1.04)^{12} \approx 1.601$

$A \approx 250 \times 1.601 = 400.25$

Answer: 400.25 TShs

18. Evaluate

$\log_3 9 \times \log_5 (1/64) \times \log_7 (1/7)$

$\log_3 9 = 2$

$\log_5 (1/64) = \log_5 (64^{-1}) = -\log_5 64 = -\log_5 (4^3) = -3 \log_5 4$

$\log_7 (1/7) = -1$

Product $= 2 \times (-3 \log_5 4) \times (-1) = 6 \log_5 4$

$\log_5 4 \approx 0.861$

$6 \times 0.861 \approx 5.166$

Answer: 5.166 (approximate)

19. If y varies inversely as $\sqrt{x}$, and x is multiplied by n

$y = k / \sqrt{x}$

New $x = nx$, new $y = k / \sqrt{(nx)} = k / (\sqrt{n}\sqrt{x}) = y / \sqrt{n}$

Ratio $= y : (y / \sqrt{n}) = \sqrt{n} : 1$

Answer: $\sqrt{n} : 1$

20. A sphere of diameter 4 cm is beaten out into a circular sheet

Surface area of sphere $= 4\pi r^2$, $r = 2$ cm

$4\pi(2)^2 = 16\pi$ cm²

Area of sheet $= \pi R^2$, thickness $= 0.03$ cm

Volume conserved: $16\pi \times 0.03 = (4/3)\pi(2)^3$

$16\pi \times 0.03 = (32/3)\pi$ (incorrect, recheck)

Sphere surface area = sheet area:

$16\pi = \pi R^2 \rightarrow R^2 = 16 \rightarrow R = 4$ cm

Answer: 4 cm

Section B: Solve 4 questions (21-23)

21. The fourth, fifth and sixth terms of the series

$T_4 = 2x + 10$, $T_5 = 4x - 4$, $T_6 = 8x + 40$

$T_5 - T_4 = T_6 - T_5$

$(4x - 4) - (2x + 10) = (8x + 40) - (4x - 4)$

$2x - 14 = 4x + 44$

$-2x = 58 \rightarrow x = -29$

$T_4 = 2(-29) + 10 = -48$

$T_1 = T_4 - 3d = -48 - 3(-14) = -6$

(i) x

Answer: $x = -29$

(ii) Arithmetic Progression

$d = T_5 - T_4 = (4(-29) - 4) - (-48) = -120 + 48 = -72$

$T_1 = -6$, $d = -72$

$S_{10} = (10/2)[2(-6) + (9)(-72)] = 5(-12 - 648) = -3300$

Answer: $S_{10} = -3300$

22. (a) Given that P, Q, and R

P = (8, 10), Q = (6, 20), R = (16, 0)

PQ = (6 - 8, 20 - 10) = (-2, 10)

PR = (16 - 8, 0 - 10) = (8, -10)

Dot product = (-2)(8) + (10)(-10) = -16 - 100 = $-116 \neq 0$ (not perpendicular)

Answer: Not perpendicular

(b) The resultant of two vectors

P = (a, b), Q = (16, -8), R = (12, -5)

P + Q = R

(a + 16, b - 8) = (12, -5)

$a + 16 = 12 \rightarrow a = -4$

$b - 8 = -5 \rightarrow b = 3$

P = (-4, 3)

R = (12, -5)

Magnitude = $\sqrt{(12^2 + (-5)^2)} = \sqrt{169} = 13$

Direction: $\tan \theta = -5/12 \rightarrow \theta \approx -22.6°$

Answer: Magnitude = 13, Direction $\approx -22.6°$

23. (a) Find the matrix B

A = [7 5; 4 3], C = [1 -1; 1 1]

AB = C

$B = A^{-1}C$

det A = 7×3 - 5×4 = 1

$A^{-1} = [3\ -5; -4\ 7]$

$B = [3\ -5; -4\ 7]\ [1\ -1; 1\ 1] = [-2\ -8; 3\ 11]$

Answer: $B = [-2\ -8; 3\ 11]$

(b) Given that $A = [0\ 2x; x\ 0]$

$|A| = -8 \rightarrow 0 - (2x)(x) = -8$

$-2x^2 = -8 \rightarrow x^2 = 4 \rightarrow x = \pm 2$

Answer: $x = \pm 2$

24. (a) Given $N = \{x: 1 \leq x \leq 20\}$

$N = \{1, 2, ..., 20\}$

(i) $A = \{x: x \text{ is a multiple of } 3\}$

$A = \{3, 6, 9, 12, 15, 18\}$

Answer: $A = \{3, 6, 9, 12, 15, 18\}$

(ii) $B = \{x: x \text{ is a multiple of } 4\}$

$B = \{4, 8, 12, 16, 20\}$

Answer: $B = \{4, 8, 12, 16, 20\}$

(iii) A'

$A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20\}$

Answer: $A' = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20\}$

(iv) B'

$B' = \{1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19\}$

Answer: $B' = \{1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 17, 18, 19\}$

(v) $(A \cup B)'$

$A \cup B = \{3, 4, 6, 8, 9, 12, 15, 16, 18, 20\}$

$(A \cup B)' = \{1, 2, 5, 7, 10, 11, 13, 14, 17, 19\}$

Answer: $(A \cup B)' = \{1, 2, 5, 7, 10, 11, 13, 14, 17, 19\}$

(vi) $A' \cap B'$

$A' \cap B' = \{1, 2, 5, 7, 10, 11, 13, 14, 17, 19\}$

Answer: $A' \cap B' = \{1, 2, 5, 7, 10, 11, 13, 14, 17, 19\}$

(b) If $D = \{x: 80 < x \leq 100\}$

$D = \{81, 82, ..., 100\}$, $n(D) = 20$

(i) divisible by 7

Multiples of 7: 84, 91, 98 → 3 numbers

$P = 3/20$

Answer: 3/20

(ii) a prime number

Primes: 83, 89, 97 → 3 numbers

$P = 3/20$

Answer: 3/20

(iii) an odd number

Odd numbers: 81, 83, 85, 87, 89, 91, 93, 95, 97, 99 → 10 numbers

$P = 10/20 = 1/2$

Answer: 1/2

25. (a) Sketch the graph of the function

$f(x) = -1 + |x|$

(i) the domain

Domain: $\mathbb{R}$

Answer: $\mathbb{R}$

(ii) the range

$f(x) = -1 + |x|$, $|x| \geq 0 \rightarrow f(x) \geq -1$

Range: $[-1, \infty)$

Answer: $[-1, \infty)$

(b) Given that $g(x) = 3 + 1/x$

(i) $g^{-1}(6)$

$g(x) = 6 \rightarrow 3 + 1/x = 6$

$1/x = 3 \rightarrow x = 1/3$

$g^{-1}(6) = 1/3$

Answer: 1/3

(ii) $g^{-1}(-1)$

$3 + 1/x = -1$

$1/x = -4 \rightarrow x = -1/4$

$g^{-1}(-1) = -1/4$

Answer: -1/4

(iii) $g^{-1}(0)$

$3 + 1/x = 0$

$1/x = -3 \rightarrow x = -1/3$

$g^{-1}(0) = -1/3$

Answer: -1/3

(iv) $g^{-1}(a)$

$g(x) = a \rightarrow 3 + 1/x = a$

$1/x = a - 3 \rightarrow x = 1/(a - 3)$

$g^{-1}(a) = 1/(a - 3)$

Answer: 1/(a - 3)

26. (a) In a class of thirty pupils

30 pupils, 3 prizes (distinct)

Number of ways = $30 \times 29 \times 28 = 24360$

Answer: 24360

(b) A girl has two jackets, four nice blouses, and three pairs of good shoes

Outfits = $2 \times 4 \times 3 = 24$

Answer: 24

(c) A boy has five different flags

Arrange 5 flags vertically: 5! = 120

Answer: 120