THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

041 BASIC MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 1997**

Instructions

1. This paper consists of Section A and B.

2. Answer all questions in section A and any four questions in section B.

1. (a) (i) Given that $M = 4 \times 10^3$, express $M/4$ in standard form

$M = 4 \times 10^3$

$M/4 = (4 \times 10^3) / 4 = 10^3 = 1 \times 10^3$

Answer: 1×10^3

(ii) Express 0.63 in the form a/b where a and b are integers such that $b \neq 0$

$0.63 = 63/100$

Answer: 63/100

(c) If $\log p = 1.813$ and $\log q = 2.513$, find the value of pq^2

$\log p = 1.813 \rightarrow p = 10^{1.813} \approx 65$

$\log q = 2.513 \rightarrow q = 10^{2.513} \approx 326$

$q^2 \approx 326^2 \approx 106276$

$pq^2 \approx 65 \times 106276 \approx 6.908 \times 10^6$

Answer: 6.908×10^6 (approximate)

2. (a) Make x the subject of the formula in the equation

$y = (cx + d)/(x - 2)$

$y(x - 2) = cx + d$

$yx - 2y = cx + d$

$yx - cx = 2y + d$

$x(y - c) = 2y + d$

$x = (2y + d) / (y - c)$

Answer: $x = (2y + d) / (y - c)$

(b) If $a + b = (a^2 - 2b)$, find

(i) 3 + 2

$a = 3, b = 2$

$3 + 2 = 3^2 - 2(2) = 9 - 4 = 5$

Answer: 5

(ii) $4 + 2$

$a = 4, b = 2$

$4 + 2 = 4^2 - 2(2) = 16 - 4 = 12$

Answer: 12

3. (a) The line passing through the point A(k, 4) and B(3, 2k) is parallel to the line $y = x + 0$

Slope of $y = x + 0$ is 1

Slope of AB $= (2k - 4) / (3 - k) = 1$

$2k - 4 = 3 - k$

$3k = 7 \rightarrow k = 7/3$

A(7/3, 4), B(3, 14/3)

Answer: $k = 7/3$

(b) A function f is defined by $f(x) = |x - 2|$

(i) Evaluate f(3)

$f(3) = |3 - 2| = 1$

Answer: 1

(ii) Find x if $f(x) = 6$

$|x - 2| = 6$

$x - 2 = 6 \rightarrow x = 8$

$x - 2 = -6 \rightarrow x = -4$

Answer: $x = 8, x = -4$

4. (a) Factorise $(1000)^3 - (999)^3$ and hence find the exact value

$(1000)^3 - (999)^3 = (1000 - 999)(1000^2 + 1000 \times 999 + 999^2)$

$= 1 \times (1000000 + 999000 + 998001) = 2997001$

Answer: 2997001

(b) If $\tan A = 2/3$ where A is an acute angle

(i) sin A

$\tan A = 2/3 \rightarrow \sin A / \cos A = 2/3$

$\sin^2 A + \cos^2 A = 1$

Let $\sin A = 2k$, $\cos A = 3k$

$(2k)^2 + (3k)^2 = 1 \rightarrow 13k^2 = 1 \rightarrow k = 1/\sqrt{13}$

$\sin A = 2/\sqrt{13}$

Answer: $2/\sqrt{13}$

(ii) cos A

$\cos A = 3/\sqrt{13}$

Answer: $3/\sqrt{13}$

(c) Rationalise the denominator of $2\sqrt{3}/\sqrt{2}$

$2\sqrt{3} / \sqrt{2} \times (\sqrt{2} / \sqrt{2}) = 2\sqrt{6} / 2 = \sqrt{6}$

Answer: $\sqrt{6}$

5. (a) In the figure drawn here under PQ = 135 mm

PQ = 135 mm, PQ shorter than 135 mm → contradiction

Assume PQ = 135 mm, PR = 6 cm = 60 mm, QR = 9 cm = 90 mm

ΔPQR: $PQ^2 = PR^2 + QR^2 \rightarrow 135^2 = 60^2 + 90^2 \rightarrow 18225 = 12600$ (not right triangle)

Recheck: PQ = 15 cm = 150 mm, PR = 6 cm, QR = 9 cm

$150^2 = 6^2 + 9^2 \rightarrow 22500 \neq 117$ (not right triangle)

Assume typo: Find PQ if PR = 6 cm, QR = 9 cm, right angle at R

$PQ = \sqrt{(6^2 + 9^2)} = \sqrt{117} \approx 10.82$ cm

PQ shorter than 10.82 cm → impossible (hypotenuse longest)

Answer: Recheck problem

6. (a) The distance between two villages P and Q is 160 km

P at 50 km/h, Q at 60 km/h, meet after x hours

$50x + 60x = 160$

$110x = 160 \rightarrow x = 16/11$ hrs

Distance from P = $50 \times 16/11 = 800/11 \approx 72.73$ km

Answer: 72.73 km from P

(b) Find x, y, and z

$P(x) = x^4 - 6x + 1$ divided by $3x + 6 \rightarrow x = -2$

$P(-2) = (-2)^4 - 6(-2) + 1 = 16 + 12 + 1 = 29$

Recheck: Problem likely from earlier context

Assume x, y, z from vectors or equations (not provided)

Answer: Recheck problem

(c) Find x, y given

[x; y] = [2 -1; 0 5] [2; 6]

[2 -1; 0 5] [2; 6] = [4 - 6; 0 + 30] = [-2; 30]

x = -2, y = 30

Answer: x = -2, y = 30

7. (a) Given p = (G, 1), q = (-1, 2), r = (1, 3), and g = (1, 1)

p = (G, 1) = (1, 1) → G = 1

Find scalars: $p + q = g \rightarrow (1, 1) + (-1, 2) = (0, 3) \neq (1, 1)$

Recheck: $p + q + r = g \rightarrow (1, 1) + (-1, 2) + (1, 3) = (1, 6) \neq (1, 1)$

Assume: ap + bq + cr = g

a(1, 1) + b(-1, 2) + c(1, 3) = (1, 1)

a - b + c = 1

a + 2b + 3c = 1

Need third equation or recheck problem

Answer: Recheck problem

8. (a) If A is the set of prime factors of 42 and B is the set of prime factors of 330

A = {2, 3, 7}

B = {2, 3, 5, 11}

$n(A \cap B) = \{2, 3\} = 2$

Answer: 2

(b) An integer is selected at random

Integers 50 to 60: 50, 51, ..., 60 (11 numbers)

Prime: 53, 59 → 2 numbers

Multiple of 4: 52, 56, 60 → 3 numbers

Square: None

P(prime) = 2/11

P(multiple of 4) = 3/11

P(square) = 0

Answer: (i) 2/11, (ii) 3/11, (iii) 0, (iv) 0

9. The line defined by the equation $x + y = 12$ crosses the y-axis at A and meets the line $x = 6$ at B

$x + y = 12$

A (y-axis, $x = 0$): $0 + y = 12 \rightarrow A = (0, 12)$

B ($x = 6$): $6 + y = 12 \rightarrow y = 6 \rightarrow B = (6, 6)$

C = (0, 3), M = midpoint of AB

(a) the co-ordinates of the A, B, and M

A = (0, 12), B = (6, 6)

M = ((0 + 6)/2, (12 + 6)/2) = (3, 9)

Answer: A = (0, 12), B = (6, 6), M = (3, 9)

(b) the co-ordinates of the line AB

Equation of AB: Slope = (6 - 12) / (6 - 0) = -1

$y - 12 = -1(x - 0) \rightarrow y = -x + 12 \rightarrow x + y = 12$

Answer: $x + y = 12$

(c) the equation of the line which is perpendicular to the line AB passing through M

Slope of AB = -1, perpendicular slope = 1

M = (3, 9)

$y - 9 = 1(x - 3) \rightarrow y = x + 6$

Answer: $y = x + 6$

10. (a) If $f(x) = (x + 2)/(x^2 - x - 6)$, find the values of x for which the function is not defined

$x^2 - x - 6 = (x - 3)(x + 2)$

Not defined when $x^2 - x - 6 = 0 \rightarrow x = 3, x = -2$

Answer: $x = 3, x = -2$

(b) Given $f(x) = [-3 \text{ when } x < -2; 1 \text{ when } -2 \leq x < 1; 4 \text{ when } x \geq 1]$

(i) Sketch the graph of f(x)

(ii) State the domain and range of f(x)

Domain: $\mathbb{R}$

Range: {-3, 1, 4}

Answer: Domain: $\mathbb{R}$, Range: {-3, 1, 4}

(iii) Is f(x) a one to one function?

No, e.g., f(-3) = f(-4) = -3

Answer: No

11. A girl wishes to buy skirts for herself

White skirt: 2400, Coloured skirt: 3000

Budget: 48000

Maximize number of skirts: $2x + 3y \leq 48$

$x \geq 1, y \geq 4$ (at least 1 white, 4 coloured)

Vertices:

(1, 4): 1 + 4 = 5

(12, 4): 12 + 4 = 16

(6, 12): 6 + 12 = 18

(1, 15): 1 + 15 = 16

Maximum at (6, 12): 18 skirts

Answer: 18 skirts

12. The daily wages of one hundred men are distributed as shown below

Wage in TShs. x 1000 | 3.0 - 3.4 | 3.5 - 3.9 | 4.0 - 4.4 | 4.5 - 4.9 | 5.0 - 5.4 | 5.5 - 5.9 | 6.0 - 6.4 | 6.5 - 6.9

Number of men | 4 | 6 | 10 | 14 | x | 20 | 14 | 6

(a) Find x

Total = 100

4 + 6 + 10 + 14 + x + 20 + 14 + 6 = 100

x = 100 - 74 = 26

Answer: x = 26

(b) Calculate the daily mean wage of the 100 men

Midpoints: 3.2, 3.7, 4.2, 4.7, 5.2, 5.7, 6.2, 6.7 (× 1000 TShs)

Mean = (4×3.2 + 6×3.7 + 10×4.2 + 14×4.7 + 26×5.2 + 20×5.7 + 14×6.2 + 6×6.7) / 100

= (12.8 + 22.2 + 42 + 65.8 + 135.2 + 114 + 86.8 + 40.2) / 100

= 519 / 100 = 5.19 × 1000 = 5190 TShs

Answer: 5190 TShs

(c) Draw a histogram to represent this data

13. (a) What is a linear transformation?

A transformation T: $\mathbb{R}^n \rightarrow \mathbb{R}^m$ is linear if T(u + v) = T(u) + T(v) and T(ku) = kT(u) for all vectors u, v and scalar k.

Answer: Defined

(b) Is a translation a linear transformation?

No, a translation T(x, y) = (x + a, y + b) does not satisfy T(0) = 0 unless a = b = 0.

Answer: No

(c) A translation takes the point (5, 8) to (12, -4)

Translation vector: (12 - 5, -4 - 8) = (7, -12)

$(8, 5) \rightarrow (8 + 7, 5 - 12) = (15, -7)$

Answer: (15, -7)

(d) A linear transformation T maps the point (x, y) onto (x', y')

$x' = x + y$, $y' = -x + 2y$

$T = [1\ 1; -1\ 2]$

Determinant = $1 \times 2 - 1 \times (-1) = 3$

$T^{-1} = (1/3)\ [2\ -1; 1\ 1]$

Answer: $T = [1\ 1; -1\ 2]$, $T^{-1} = (1/3)\ [2\ -1; 1\ 1]$