THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2

ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Time: 3 Hours **Monday 13 May 2002 p.m.**

Instructions

1. This paper consists of sections A and B
2. Answer **ALL** questions in section A and any **FOUR** (4) questions from section B.
3. All necessary steps in answering each question must be shown clearly.
4. All answers must be written in the answer booklet provided.
5. Mathematical tables, mathematical formulae, slide rules and unprogrammable pocket calculators may be used.
6. Cellular phones are not allowed in the examination room.
7. Write your Examination Number on every page of your answer booklet.

SECTION A (60 Marks)

Answer ALL questions in this section showing ALL necessary steps and answers.

1. (a) Use four figure tables to find the value of $\sqrt{\frac{\sqrt{19}\, e^2 \ln 3}{\sqrt{2}}}$ correct to 3 decimal places. (04 marks)

   (b) By using a non programmable scientific calculator, find the value in (a) correct to 10 significant figures. (02 marks)

2. (a) Given that $S_1 = (p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$ and $S_2 = \sim(\sim q \rightarrow \sim p) \rightarrow \sim(p \rightarrow q)$ use truth tables to find whether or not S_1 is equivalent to S_2. (02½ marks)

   (b) Write the statement $(\sim p \rightarrow q) \rightarrow (\sim q \rightarrow \sim p)$ using the connective $\sim$ and $\vee$, and prepare a truth table for the resulting disjunction. (03½ marks)

3. (a) Two curves $y = 2x^2 - 3$ and $y = x^2 - 5x + 3$ intersect at two points. P is one of them and is in the fourth quadrant. Find the tangent of the acute angle between these curves at the point P. (03 marks).

   (b) A point Q moves such that its distance from the point (5, 3) is equal to twice its distance from the line $x = 2$. Find the equation of the locus. (03 marks)

4. (a) Prove that $\frac{\sin 8\theta - \sin 6\theta \cos 3\theta}{\cos 2\theta \cos \theta - \sin 3\theta \sin 4\theta} = \tan 2\theta$ (03 marks)

   (b) Solve the equation $5\cos x - 2\sin x = 2$ for $-180° \leq x \leq 180°$ using the substitution $t = \tan \frac{1}{2} x$ (03 marks)

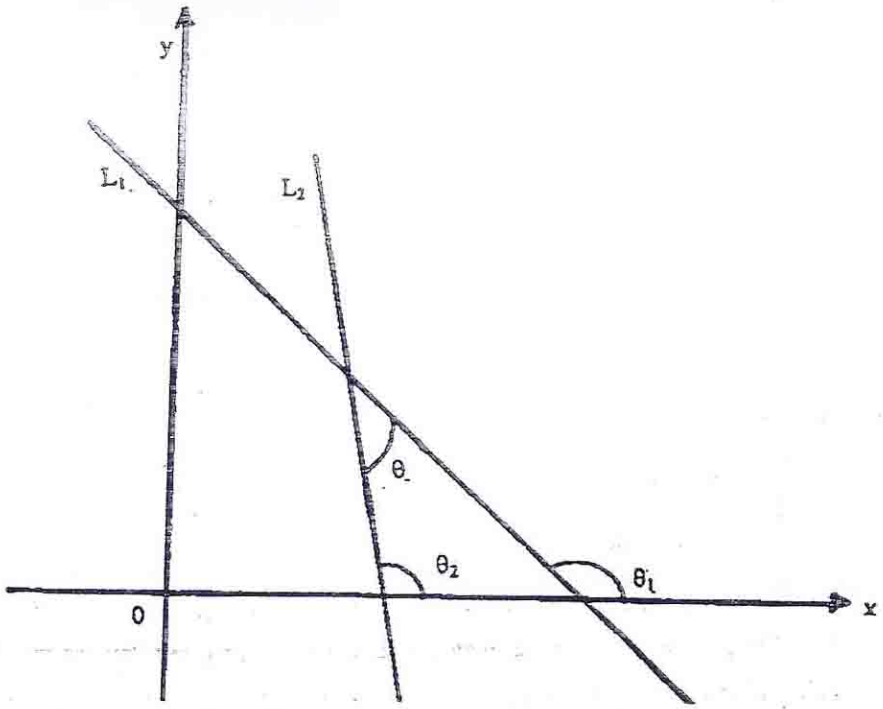
5. (a) The first term of a geometric series is 27 and its common ratio is $\frac{4}{3}$. Find the least number of terms so that the series can have its sum exceeding 550. (02 marks)

   (b) Express the series $1 + 4 + 7 + 10 + 13 + \text{---} + 298$ in the form $\sum_{r=1}^{n} f(r)$. (02 marks)

   (c) Use the method of induction to prove that $\sum_{r=1}^{n} 2^{r-1} = 2^n - 1$ where $n = 1, 2, 3, \text{---}$. (02 marks)

6. (a) The four points A(-2, -1), B(4, 3), C(x, y) and D(0, -4) are the vertices of a parallelogram ABCD. Find the values of x and y. (02 marks)

(b) (i) Two straight lines L_1 and L_2 are drawn in the xy plane as shown in the diagram below



Prove that $\theta = \tan^{-1} \dfrac{m_1 - m_2}{1 + m_1 m_2}$ where m_1 and m_2 are the gradients of L_1 and L_2 respectively.

(ii) Find the acute angle between the lines whose equations are $y + 2x = 3$ and $y + x = 0$. (04 marks)

7. Use the hyperbolic substitution to find $\int \dfrac{dx}{x^2 (x^2 + 4)^{\frac{1}{2}}}$. (06 marks)

8. The heights in cm of 70 pupils in a certain class were distributed as follows:

-119	120-129	130-139	140-149	150-159	160-169	170-179
0	7	10	y	18	11	9

(a) Find the value of y (01 mark)

(b) Compute the variance and mean of the data. (05 marks)

9. (a) Four digit numbers are formed from the digits 0, 1, 2, 3, 6, 9 without repeating a digit. How many different numbers will be formed? (02 marks)

(b) One four digit number in 9 (a) is chosen at random, what is the probability that:

(i) it is greater than 4,000

(ii) it is between 2000 and 6000 and divisible by 5? (04 marks)

10. (a) Given that $\alpha = 1 + 3i$ is a root of the equation $z^2 - (p + 2i)\,z + q\,(1 + i) = 0$ and that p and q are real numbers, determine the values of p and q. (3 marks)

(b) If z is a complex number, find the locus represented by the equation

$$\left|\frac{z-1}{z+1}\right| = 2.$$ (3 marks)

SECTION B (40 Marks)

Answer FOUR (4) questions from this section showing all necessary steps and answers.

11. (a) Find the point of intersection of the line $\frac{x-1}{4} = \frac{y+2}{3} = \frac{z-1}{2}$ with the plane $2x - y + 2z = 5$ (3 marks)

(b) Given the vectors:

$\mathbf{a} = 2m\mathbf{i} - 2\mathbf{j} + \mathbf{k}$
$\mathbf{b} = \mathbf{i} + 2n\mathbf{j} - 3\mathbf{k}$
$\mathbf{c} = 3\mathbf{i} - \mathbf{j} + 2r\mathbf{k}$

(i) Find the values of m, n and r if the algebraic sum of the vectors is a null vector.

(ii) Using the results in (i) calculate the area of the plane formed by vectors a and b.

(iii) Find the volume of the parallelopiped as a function of m, n and r whose sides are the vectors given above. (07 marks)

12. (a) If the matrix $A = \begin{pmatrix} 3 & -2 & -2 \\ 2 & 3 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ verify that $\det A = \det A^T$. (3 marks)

(b) Find the adjoint of the matrix in (a) and use it to solve the following system of equations:

$3x - 2y - 2z = 1$

$2x + 3y - z = 9$

$x - y + 3z = -4$ (07 marks)

13. (a) Define the coordinates of p(x,y) in polar form and use it to write the following equations in most simplified polar form:

(i) $(x-a)^2 + (y-b)^2 = c^2$

(ii) $ax + by + c = 0.$ (04½ marks)

(b) Transform equations (i) and (ii) below into cartesian coordinates and complete them as instructed:

(i) $r^2 - 4r\cos\theta + 4 = 0$ and show that it has a unique solution.

(ii) $\sin^2\theta + 8\cos^2\theta = 4$ and verify that the curve is the result of the product of two straight lines passing through the origin. (05½ marks)

14. (a) Starting from the definition of the hyperbolic function cosh x, show that $\cosh(m+1)x - \cosh(m-1)x = 2\sinh mx$ (03 marks)

(b) Evaluate $\int \frac{dx}{\sqrt{x^2 - 6x + 13}}$ using a hyperbolic function substitution. (03 marks)

(c) Find the coordinates of the minimum point of the catenary $y = a\cosh\frac{x}{a}$. (04 marks)

15. (a) Find:

(i) $\int xe^{2x}dx$.

(ii) $\int_0^{\frac{1}{2}} \frac{x^2 dx}{\sqrt{1-x^2}}$. (05 marks)

(b) The area under a straight line $y = 2x$ bounded by x axis, $x = 1$ and $x = h$ is rotated about the x axis.

(i) Sketch and identify the nature of the solid generated.

(ii) Find the equation of the volume so formed as a function of h. (05 marks)

16. (a) Simplify (i) $p \rightarrow (\sim p \rightarrow q)$

(ii) $p \wedge (q \nrightarrow \sim p)$ (04 marks)

(b) Write the converse and construct the truth table of the contrapositive of the conditional $(p \wedge q) \rightarrow \sim q$ (02 marks)

(c) Test the validity of the argument whose premises are $p \rightarrow (\sim p \rightarrow q)$, $q \nrightarrow \sim p$ and p and whose conclusion is $\sim q$. (04 marks)