THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2

ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Time: 3 Hours *Wednesday 14 May 2003 p.m.*

Instructions

1. This paper consists of sections A and B.
2. Answer **ALL** questions in section A and **FOUR (4)** questions from section B.
3. All necessary steps in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae, slide rules and nonprogrammable pocket calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your Examination Number on every page of your answer booklet(s).

This paper consists of 5 printed pages.

SECTION A (60 Marks)

Answer ALL questions in this section showing ALL necessary steps and answers.

1. Using a scientific calculator find

   (a) $\tan^{-1}(-\frac{4}{5}) + \sinh^{-1}(\frac{2}{3})$ (01½ marks)

   (b) $\log_6 9 - \ln(\frac{3}{8})$ (01½ marks)

   (c) the mean and standard deviation of the distribution given in the table below:

x	1.95	3.95	5.95	7.95	9.95	11.95
f	9	39	45	42	11	4

(03 marks)

2. (a) Given the statement: "If two vectors are orthogonal then their scalar (dot) product is zero". Write its

   (i) inverse (ii) converse (iii) contrapositive. (03 marks)

   (b) Determine the truth values of the following sentences:

   (i) Either $2<1$ or $2-7 \neq -5$.

   (ii) If $2+1 = 10$ then $12>10$. (03 marks)

3. (a) The end point coordinates of a line segment AB are $A(x_1, y_1)$ and $B(x_2, y_2)$. Prove that the coordinates of the point $p(x, y)$ dividing the line segment AB internally in the ratio $\lambda : \mu$ are given by

$$p(x, 1) = \left(\frac{\mu x_1 + \lambda x_2}{\mu + \lambda} , \frac{\mu y_1 + \lambda y_2}{\mu + \lambda} \right)$$ (03 marks)

   (b) Find the coordinates of a point dividing the line segment joining Q(-3, 6) and R(6, 0) internally in the ratio 2 : 1. (03 marks)

4. (a) Find the value of angle A in triangle ABC which is such that: $a = 10$, $b = 12$, $c = 9$. (02 marks)

   (b) Simplify $\sin^{-1}(\cos x)$ (02 marks)

   (c) Evaluate $\tan\left(\cos^{-1}(\frac{1}{2})\right)$ without using calculators or tables. (02 marks)

5. (a) If p, q, r are the roots of the equation $2x^3 + 3x^2 - x - 4 = 0$, form the equation whose roots are p^2, q^2, r^2.

   (b) Find the set of real values of x for which $|3 - 2x| < |4 + x|$. (03 marks).

6. (a) Describe, by eliminating θ, the curve represented by the equation

   $x = 4\cos\theta$ and $y = 3\sin\theta$. (03 marks)

(b) Show that the locus of a point p which moves such that its distance from the point (ae, o) is e times its distance from the line $x = \frac{a}{e}$ is the curve $\frac{x^2}{a^2} + \frac{y^2}{a^2(1-e^2)} = 1$ (03marks)

7. (a) By using its logarithmic form, show that the function $\cosh^{-1} x$ is double valued. (04 marks)

(b) If $x = \frac{1}{2}\ln 3$, find (i) $\cosh x$ (ii) $\tanh x$. (02 marks)

8. The frequency distribution below shows the number of students at Nairobi University according to their heights:

Classes (height in cm)	Freq. (f)
60 – 62	5
63 – 65	18
66 – 68	42
69 – 71	27
72 – 74	8
75 – 77	12
78 – 80	16

Using the data above, find

(a) the mean (02 marks)

(b) semi-interquartile range. (04 marks)

9. (a) The probability that Hamisi will pass this paper is 0.85 and that Amani will pass is 0.75. Find he probability that

(i) both will pass (ii) Hamisi or Amani will pass. (03 marks)

(b) A box contains 9 blue and 11 red balls. Three balls are drawn at random from the box and without replacement. Find the probability that

(i) all three are of the same colour

(ii) one of the balls is red. (03 marks).

10. (a) Solve $z^3 - 1 = 0$ giving your solution in polar form. (03 marks)

(b) (i) Show that

$$\text{Sin } n\theta = \frac{1}{2i}\left(z^n - \frac{1}{z}n\right)$$ (01½ marks)

(ii) Express $\sin^3\theta$ in terms of multiple angles of θ. (01½ marks)

SECTION B (40 marks)

Answer FOUR (4) questions from this section showing all necessary steps and answers.

11. (a) Find the shortest distance from the point B(–1, –1, 1) to the line $r = 2j - 3k + \lambda(2i - j + 2k)$ (04 marks)

(b) The equation of the plane is parametrically written as

$$r = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} + \cup \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}.$$

Find the cartesian equation of the plane. (02½ marks)

(c) Show that the lines

$r = i + \lambda(6i + 2j - 3k)$ and $r_1 = i_0 + j + k + \cup(-2i + j - 2k)$ are skew. (03½ marks)

12. (a) Solve the following system of equations by using Cramer's rule

$$\begin{cases} 2x + 3y - z = -7 \\ -3x + y + 2z = 1 \\ 3x - 4y - 4z = -1 \end{cases}$$ (04 marks)

(b) Prove by using partial fractions that

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \ldots \frac{1}{n(n+1)} = \frac{n}{n+1}$$ (04 marks)

(c) Find the value of $(1.023)^{\frac{1}{3}}$ without using tables or calculators to seven significant figures. (02 marks)

13. (a) (i) Express the equation $(x^2 + y^2)^2 = x^2 - y^2$ in polar form.

(ii) Find the cartesian equation of the locus given by $r = 16 \cot\theta \operatorname{cosec}\theta$. (05 marks)

(b) Find the eccentricity and coordinates of the foci of the ellipse $4x^2 + 9y^2 = 36$.

14. (a) Find $\frac{dy}{dx}$ and simplify your answer, given that $y = \frac{e^x - 1}{e^x + 1}$ (05 marks)

(b) Evaluate the following integral correct to three significant figures.

$$\int_1^2 \frac{1}{\sqrt{x^2 + 4x + 8}}\, dx$$ (05 marks)

15. (a) "Is it true that girls perform poorly in science subjects? " Is this a mathematical statement? Why? (02 marks)

(b) Given that a sentence has the truth table below, write down its expression in a simplified form.

P∧q	P∧~q	~P∧q	?
T	T	F	T
F	T	F	T
F	F	T	T
F	F	F	F

(05 marks)

(c) Test the validity of the argument $P \rightarrow q$, $q \vee \sim r \quad \therefore \sim r \rightarrow \sim P$ (03 marks)

16 (a) (i) If $x > 1$, prove that

$$\frac{1}{2}\log_e\left(\frac{x+1}{x-1}\right) = \frac{1}{x} + \frac{1}{3x^3} + \frac{1}{5x^5} + \ldots$$ (03 marks)

(ii) Use the result in (i) above to calculate $\log_e 2$ to three decimal places. (02 marks)

(b) Integrate the following with respect to x:

$$f(x) = \frac{5x+7}{x^2+4x+8}dx$$ (05 marks)