THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1

(For Both School and Private Candidates)

Time: 3 Hours **Tuesday 04 May 2004 a.m.**

Instructions

1. This paper consists of sections A and B.
2. Answer **all** questions in section A and **four (4)** questions from section B.
3. All work done in answering each question must be clearly shown.
4. Mathematical tables, mathematical formulae, slide rules and unprogrammable scientific pocket calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your Examination Number on every page of your answer booklet(s).

This paper consists of 4 printed pages.

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SECTION A (60 marks)

Answer all questions in this section showing all necessary steps and answers.

1. (a) Use the appropriate laws of set algebra to simplify $(A \cup B)' \cap A$. **(02 marks)**

   (b) An investigator was promised to be paid shs. 100 per person she interviewed about their likes and dislikes on a drink for lunch. She reported that 252 responded positively to coffee, 210 liked tea, 300 liked soda, 80 liked tea and soda, 60 liked tea and coffee, 110 liked coffee and soda, 50 liked all three while 120 people said they did not like any drink. How much should the investigator be paid?
   (04 marks)

2. (a) A point moves in such a way that its distance from the x-axis is always ⅕ its distance from the origin. Find the equation of its path. **(02 marks)**

   (b) The perpendicular line from point A (–1, 2) to the straight line $2y + 3x - 14 = 0$ intersect the line at point B. If the perpendicular is extended to C in such a way that AB = ½ BC determine the coordinates of C.
   (04 marks)

3. (a) Find the range of the function $f: x \rightarrow |x - 1|$ whose domain is given by $|x| \leq 3$. **(02 marks)**

   (b) The function $f: x \rightarrow \frac{a}{x} + b$ is such that $f(2) = 2$ and $f(-1) = -1$;

   (i) find the value of a and b;

   (ii) sketch the graph of f.
   (04 marks)

4. (a) Using the standard results for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r$ obtain an expression for $\sum_{r=1}^{n} r(r+1)$ and simplify your answer as far as possible. **(02 marks)**

   (b) The sum of the first t terms of a geometric progression is 9 and the sum of the first four terms is 45. Calculate the two possible values of the fifth term.
   (04 marks)

5. (a) Prove the following identities:

   (i) $\frac{\cos 2x}{\cos x + \sin x} = \cos x - \sin x$

   (ii) $\frac{\cos x}{\sin y} - \frac{\sin x}{\cos y} = \frac{2\cos(x+y)}{\sin 2y}$
   (02 marks)

   (b) Find the values of x, for $0 \leq x \leq 180°$ which will satisfy the equation.

   (i) $\tan 4x = 1$

   (ii) $\sin 3x = 0$
   (04 marks)

6. (a) If $\sin(x + \alpha) = \cos(x - \beta)$, find $\tan x$ in terms of cosines and sines of α and β. Hence find $\tan x$ if $\sin(x + 60°) = \cos(x - 45°)$. Leave your answer in surd forms but with a rational denominator.
   (03 marks)

   (b) Prove that $\frac{\sec\theta - \operatorname{cosec}\theta}{\tan\theta - \cot\theta} = \frac{\tan\theta + \cot\theta}{\sec\theta + \operatorname{cosec}\theta}$
   (03 marks)

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7. (a) If $\ln(x+y) = \tan^{-1}\left(\frac{x}{y}\right)$, find $\frac{dy}{dx}$. **(03 marks)**

(b) The pressure p and the volume v of an expanding gas are related by $pv^{1.4} = k$. If the volume increases by 0.3 %, what is the corresponding percentage change in the pressure? **(03 marks)**

8. (a) Find a unit vector in the direction of $\underline{a} = 6\underline{i} + 3\underline{j} + \underline{k}$ and hence state its length. **(02 marks)**

(b) Find the constant P such that the vectors $\underline{a} = 2\underline{i} - \underline{j} + \underline{k}$, $\underline{b} = \underline{i} + 2\underline{j} - 3\underline{k}$ and $\underline{c} = 3\underline{i} + P\underline{j} + 5\underline{k}$ are coplar vectors. **(04 marks)**

9. (a) Evaluate $\int_{-1}^{1} \frac{dx}{1-\sin x \cos x}$ **(02 marks)**

(b) Find $\int \frac{4x^2 - 2x + 3}{(x^2+2)(x-2)}\, dx$ **(04 marks)**

10. Given a set of numbers {18, 16, 13, 14, 12, 10, 11, 15, 17, 19}
Find:

(a) the median and the mode. **(1½ marks)**

(b) the mean and the standard deviation. **(02 marks)**

(c) quartile one and quartile three. **(02 marks)**

(d) the range. **(00½ mark)**

SECTION B (40 marks)

Answer **four (4)** questions from this section, showing all necessary steps and answers.

11. A cement dealer has two depots D_1 and D_2 holding 120 and 40 tons respectively. He has two customers C_1 and C_2 who have ordered 80 and 50 tons of cement respectively. C_1 is 20 km from D_1 and 40 km from D_2. C_2 is 15 km from D_1 and 30 km from D_2. Delivery costs are proportional to the distance travelled. How should he supply his customers to minimize the total transport costs? **(10 marks)**

12. (a) Express $(\sqrt{3} + i)^5$ in the form $r(\cos\theta + i\sin\theta)$. **(02 marks)**

(b) Given that z_1 and z_2 are the roots of the quadratic equation $((1+2i)z^2 + (1-5i)z + 7i - 6 = 0$, find the values of $z_1 z_2$ and $z_1 + z_2$ in the forms a + bi. **(05 marks)**

(c) Use De Moivres theorem to prove that $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$. **(03 marks)**

13. (a) Verify that the equation $x^2 - 2x - 1 = 0$ has a root lying between x = 2 and x = 3. Apply the method of bisection in four iterations to obtain an approximation of the root. **(04 marks)**

(b) Starting with $x_0 = 2$, calculate the approximation to the root of the equation in 13 (a) above correct to three decimal places with only two iterations. **(03 marks)**

(b) Apply Simpson's rule with n = 4 to obtain an approximation for the integral $I = \int_0^1 \frac{dx}{1+x}$. **(03 marks)**

$$\sum_{r=1}^{n} r^2 = \frac{n(2n+1)(n+1)}{6}$$

$$P_1V_1^{1.4} = P_2V_2^{1.4}$$

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14. (a) Find the general solution to the D.E.

(i) $\frac{dy}{dx} - y \cot x = 0.$

(ii) $\frac{dy}{dx} = \frac{2x + 6y + 3}{x + 3y - 2}.$ **(05 marks)**

(b) Form a D.E. whose solution is the function $y = Ae^{2x} + Be^{-3x}$ where A and B are constants. **(05 marks)**

15. (a) Show that $\sum_{x=0}^{N} x p(x) = np$ given that $p(x) = {}^{N}C_x p^x q^{N-x}$. **(03 marks)**

(b) A fair coin is tossed 500 times. Find the probability that the number of heads will not differ from 250 by:

(i) More than 10.

(ii) More than 30.

(07 marks)

16. A particle of mass M_2 kg rests on the surface of a smooth plane inclined at an angle α to the horizontal and is connected by alight inextensible string passing over a pulley at the top of the plane to a mass M_1 kg hanging freely. Assuming $M_1 > M_2$ show that:

(a) The acceleration of the system is given by $\frac{g(M_1 - M_2 \sin\alpha)}{M_1 + M_2}$. **(06 marks)**

(b) The tension in the string is given by $\frac{M_1 M_2 g(1+\sin\alpha)}{M_1 + M_2}$. **(04 marks)**

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