THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1

(For Both School and Private Candidates)

Time: 3 Hours

2006 February, 07 Tuesday a.m.

INSTRUCTIONS

1. This paper consists of sixteen (16) questions in sections A and B.
2. Answer all questions in section A and four (4) questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae and non-programmable calculators may be used.
5. Cellular phones are not allowed in the examination room.
6. Write your Examination Number on every page of your answer booklet(s).

ACS

This paper consists of 5 printed pages.

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SECTION A (60 marks)

Answer **all** questions in this section.

1. (a) Using the basic properties of set operations, simplify each of the following expressions:

(i) $(X \cap Y') \cup (X \cup Y')$.

(ii) $(X \cap Y') \cup (X' \cap Y) \cup (X \cap Y)$.

(2 marks)

(b) In a class of 35 students, each student takes either one or two science subjects (Physics, Chemistry and Biology). If 13 students take Chemistry, 22 students take Physics, 17 students take Biology, 6 students take both Physics and Chemistry and 3 students take both Biology and Chemistry, find the number of students who take both Biology and Physics.

(4 marks)

2. (a) Find the centre and radius of each of the circles A and B whose equations are:

$x^2 + y^2 - 16y + 32 = 0$ and $x^2 + y^2 - 18x + 2y + 32 = 0$ respectively.

(2 marks)

(b) Find the coordinates of the point of contact of the circles in 2(a) above and show that the common tangent at this point of contact passes through the origin.

(4 marks)

3. (a) For what real values of x is $\left|\frac{1}{1+2x}\right| = 1$?

(2 marks)

(b) The functions f and g are defined as follows:

$f: x \to e^{-x} \quad (x \in \mathbb{R}^+)$

$g: x \to \frac{1}{1-x} \quad (x \in \mathbb{R}, x < 1)$

(i) Give the ranges f, g and gof.

(ii) Give definitions of the inverse functions of f^{-1}, g^{-1} and $(gof)^{-1}$ in a form similar to the above definitions.

(4 marks)

4. The sum of 3n terms of the series $20 + 23 + 26 + 29 + \ldots$ is equal to the sum of 2n terms of the series $a + (a + d) + (a + 2d) + (a + 3d) + \ldots$ for all values of n. Prove that $d = \frac{27}{4}$ and find the value of a.

(6 marks)

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5. (a) Find all values of x within the interval $0 \leq x, \leq 2\pi$ which satisfy the equation $2\cos x - 3\sec x = 3\tan x$.

(3 marks)

(b) Show that $\tan^{-1}\left[\frac{3}{7}\right] + \tan^{-1}\left[\frac{5}{9}\right] = \tan^{-1}\left[\frac{31}{24}\right]$.

(3 marks)

6. A rectangular tank, open on top, have a capacity of 32 m^3. Prove that if its length is x metres, its width y metres and the area of its outer surface A m^2, then $A = xy + 64\left(\frac{1}{y} + \frac{1}{x}\right)$.

(3 marks)

Prove further that if y remains constant while x varies, the minimum of A is $(16\sqrt{y} + \frac{64}{y})\ m^2$.

(3 marks)

7. (a) Obtain the value of λ which makes the vectors $\underline{i} - \underline{j} + \underline{k}$, $2\underline{i} + \underline{j} - \underline{k}$ and $\lambda\underline{i} - \underline{j} + \lambda\underline{k}$ lie on one plane.

(3 marks)

(b) Find the cosine of the angle between vector $\underline{AB}$ and $\underline{AC}$, if A, B and C are the points $(-3, 4)$, $(3, 1)$ and $(-1, 5)$ respectively.

(3 marks)

8. (a) Prove that $\int_1^{e^2} x^2 \ln x \, dx = \frac{5e^6 + 1}{9}$.

(3 marks)

(b) Evaluate $\int_0^{\pi/2} \cos^3 x \sin^2 x \, dx$.

(3 marks)

9. (a) In a certain Mathematics examination, one student estimates that his chances of getting an A is 10 %, B is 40 %, C is 35 %, D is 10 %, E is 4 % and S is 1 %. By obtaining an A he gets 5 points, for B he gets 4 points, for C he gets 3 points, for D he gets 2 points, for E he gets 1 point and for S he gets ½ point. Find his expectation.

(4 marks)

(b) In how many different ways can the letters of the following words be arranged?

(i) NONE (ii) MINE

(2 marks)

10. The examination marks obtained by 100 candidates are distributed as follows:

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Mark	No. of candidates
0 – 19	8
20 – 29	7
30 – 39	14
40 – 49	23
50 – 59	26
60 – 69	12
70 – 79	6
80 – 89	4

By using the coding method, calculate the mean and standard deviation.

(6 marks)

SECTION B (40 marks)

Answer **four (4)** questions from this section.

11. (a) Minimize $z = -x + 2y$ subject to

$$-x + 3y \leq 10$$
$$x + y \leq 6$$
$$x - y \leq 2$$
$$x \geq 0$$
$$y \geq 0$$

(3 marks)

(b) A company makes two types of furniture: chairs and tables. The contribution of each product is shs. 1,500 per chair and shs. 2,500 per table. Both products are processed by three machines M_1, M_2 and M_3. The time required, in hours, per week on each machine is as follows:

Machine	Chair	Table	Available time
M_1	3	3	36
M_2	5	2	50
M_3	2	6	60

(i) How should the company schedule its production in order to maximize contribution?

(5 marks)

(ii) Find the maximum contribution.

(2 marks)

12. (a) Find the roots of the equation $z^3 - 8i = 0$.

(03 marks)

(b) Using Binomial theorem, expand $(\cos\beta + i\sin\beta)^4$. With the help of the De Moivres theorem, show that $\tan 4\beta = \dfrac{4\tan\beta - 4\tan^3\beta}{1 - 6\tan^2\beta + \tan^4\beta}$.

(5 marks)

(c) Find the equation in terms of x and y of the locus represented by $|z - 1| = |z - i|$.

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(2 marks)

13. (a) Given the function $f(x) = x^3 + 2x - 4$, show that it has a real root which lies between $x = 1$ and $x = 1.5$.

(3 marks)

(b) Using the function in (a) above and taking $x_o = 1.2$ as the first approximation of its root, use the Newton-Raphson method to find x_1, x_2 and x_3, giving your answer to 4 decimal places.

(5 marks)

(c) Give a reason why you think x_3 is the most correct answer.

(2 marks)

14. (a) Solve for the initial value of the problem; $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 10y = 0$, given that $y = 4$ when $x = 0$ and $\frac{dy}{dx} = 1$ when $x = 0$.

(5 marks)

(b) Solve the differential equation $x^2 \frac{dy}{dx} = y(x + y)$.

(5 marks)

15. (a) A continuous random variable x having values only between 0 and 4 has a density function given by:

$P(x) = \frac{1}{2} - rx$, where r is a real number.

Find the:

(i) value of r.
(ii) expectation of x
(iii) variance of x.
(iv) probability, $P(1 < x < 2)$.

(5 marks)

(b) The mean mark on a given final examination is 72 % and the standard deviation is 9. The top 10 % of the students are to receive A's. If the marks are normally distributed, what is the minimum mark a student must get in order to receive an A?

(5 marks)

16. An arrow is projected from the firing position A which hits the target B, the line joining A and B is horizontal and of length 78 m. The initial direction of the arrow makes an angle of 24 ° with the horizontal. Find the

(a) magnitude of the velocity of projection.
(b) time taken by the arrow to reach B from A.
(c) greatest height of the arrow above AB during its flight.

(10 marks)

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