THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2

ADVANCED MATHEMATICS 2
(For School and Private Candidates)

Time: 3 Hours

2006 February, 13 Monday, p.m.

INSTRUCTIONS

1. This paper consists of **sixteen (16)** questions in sections A and B.
2. Answer **all** questions in section A and **four (4)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

This paper consists of 6 printed pages.

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SECTION A (60 marks)

Answer all questions in this section.

1. (a) By using mathematical tables, compute:

(i) $\sqrt{0.031}$ correct to 3 decimal places.

(ii) $\dfrac{\tan 36.75^{\circ}}{\sin 60^{\circ}48'}$

(b) By using a scientific calculator, find: **(3 marks)**

(i) $e^{-\frac{2}{3}} + 12\cosh^{-1} 1.5$ correct to 6 significant figures.

(ii) $\ln \frac{2}{3} - 8^{\frac{1}{6}}$ correct to 6 significant figures.

(iii) The mean and standard deviation for the scores; 64, 44, 56, 75 and 66. **(3 marks)**

2. (a) Let A_{pq} denote $p \wedge q$ and let Np donate $\sim p$. Write the following propositions using A and N instead of $\wedge$ and $\sim$:

(i) $\sim p \wedge q$

(ii) $\sim(p \wedge \sim q)$

(iii) $(\sim p \wedge q)$

(b) Use algebra of propositions to show that $(p \wedge q) \vee [\sim r \wedge (q \wedge p] \equiv p \wedge q$. **(3 marks)**

3. Determine the image of the curve $(x-4)^2 + (y+1)^2 = 9$ under the following transformations: **(3 marks)**

(a) Translation by the arrow $(-4, 1)$.

(b) Reflection on the line $y = x$.

(c) Dilation by factor 2 in y direction.

(d) Magnification by factor c.

(e) What important property can be seen about the shape of the locus in 3. (a) above after the transformation?

(6 marks)

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4. (a) Solve the equation:

$\cos 2x + \cos x + 1 = 0$ for $0 \leq x \leq = 360°$.

(3 marks)

(b) Show that

$$\frac{\sin 2A + \cos 2A + 1}{\sin 2A + \cos 2A - 1} = \frac{\tan(45° + A)}{\tan A}$$

(3 marks)

5. (a) For what values of x is $\sin\theta = \frac{x-1}{x+1}$?

(3 marks)

(b) Find the sum of the following series and then simplify it.

$1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + n(n+1)$.

(3 marks)

6. The normal to the parabola $y^2 = 4x$ at P(1, 2) meets the x axis at G, and M is the midpoint of PG. A line through M parallel to the y axis meets the x axis at N and the parabola at Q. Prove that QN = PG.

(6 marks)

7. (a) If $\tan\theta = \sinh\theta$, prove that $\theta = \ln(\sec\theta + \tan\theta)$.

(3 marks)

(b) Show that if $a\cosh x + b\sinh x = c$ has equal roots, then $c^2 = a^2 - b^2$.

(3 marks)

8. (a) For the random variable x, show that $\text{Var}(x) = E^2(x) - [E(x)]^2$.

(3 marks)

(b) The random variable has a probability density function P(X = x) for x = 1, 2, 3 as shown in the table below:

x	1	2	3
P(x)	0.1	0.6	0.3

Find:

(i) $E(5x + 3)$.

(ii) $E(x^2)$.

(iii) $\text{Var}(5x + 3)$.

(3 marks)

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9. A and B play 12 games of chess of which 6 games are won by A, 4 games are won by B and 2 end in a draw. They agree to play a tournament consisting of 3 games. Find the probability that:

(a) A and B win alternately.

(b) 2 games end in a draw.

(c) B wins at least one game.

(6 marks)

10. (a) Express $\dfrac{1}{1+\cos\theta+i\sin\theta}$ in the form $x + iy$.

(2½ marks)

(b) Given that $x + iy = (-3-2i)^n$ where $x, y, \in \mathbb{R}$, $n \in Z$. Prove that $x^2 + y^2 = 13^n$.

(3½ marks)

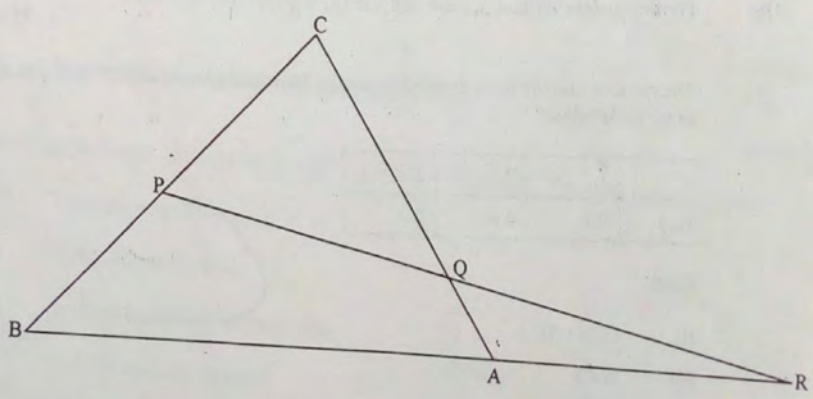
SECTION B (40 marks)

Answer **four (4)** questions from this section.

11. The vertices A, B and C of a triangle have position vectors $\underset{\sim}{a}$, $\underset{\sim}{b}$ and $\underset{\sim}{c}$ respectively relative to the origin O. Point P is on BC such that BP : PC = 3 : 1. Point Q is on AC such that CQ : QA = 2 : 3. Point R on BA produced is such that BR : AR = 2 : 1. The position vectors of P, Q and R are $\underset{\sim}{p}$, $\underset{\sim}{q}$ and $\underset{\sim}{r}$ respectively.

(a) Show that $\underset{\sim}{q}$ can be expressed in terms of $\underset{\sim}{p}$ and $\underset{\sim}{r}$, hence show that P, Q and R are collinear.

(b) State the ratio of the line segments PQ and QR.



(10 marks)

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12. (a) $(2x - y)^{20}$ is expanded as a series in descending powers of x. Find the:

(i) position and the value of the term in x^3.

(ii) position and the value of the term free from x.

(3 marks)

(b) Express $\frac{2}{n(n+2)}$ in partial fractions and deduce that

$$\frac{1}{1\times 3}+\frac{1}{2\times 4}+\frac{1}{3\times 5}+\cdots+\frac{1}{n(n+2)} = \frac{3}{4}-\frac{2n+3}{2(n+1)(n+2)}.$$

Hence find $\sum_{n=1}^{100}\frac{1}{n(n+2)}$.

(7 marks)

13. (a) Sketch the graph of $r = \sin 3\theta$ for $0° \le \theta \le 180°$.

(6 marks)

(b) Write down the equations for the asymptotes to the hyperbola $x^2 - y^2 = 1$ and hence sketch the hyperbola in the cartesian plane.

(2 marks)

(c) Transform the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ to its corresponding polar equation.

(2 marks)

14. (a) Given that $y = \tan^{-1} x$, show that when $|x| < 1$ $\tanh^{-1} x = \frac{1}{2}\ell n\frac{1+x}{1-x}$.

(5 marks)

(b) A particle is moving along a curve so that its velocity t seconds later is given by

$$v = \frac{1}{\sqrt{t^2 + 4t - 5}}.$$

Starting with $v = \frac{ds}{dt}$, find the expression for its displacement S at t seconds given that S = 0 when t = 1.

(5 marks)

15. (a) Using the laws of algebra of sets:

(i) Simplify $\sim(\sim(p \vee q) \vee (\sim p \wedge q))$

(ii) Verify that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

(4½ marks)

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(b) Construct an electrical network for the proposition $(p \vee q) \wedge [(r \vee s) \wedge w]$. (1½ marks)

(c) Draw the simplest network diagram corresponding to the statement of the following truth table.

p	q	r	Statement
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

(4 marks)

16. (a) Find

(i) $\int x^3 e^{2x} dx$.

(ii) $\int \frac{dx}{x^2\sqrt{4+x^2}}$.

(4 marks)

(b) Evaluate $\int_{\frac{1}{2}}^{1} \frac{\sin x}{x} dx$ accurate to 6 decimal places. (6 marks)

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