THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1
(For Both School and Private Candidates)

Time: 3 Hours **Monday, 11th February 2013 a.m.**

Instructions

1. This paper consists of **ten (10)** questions, each carrying **ten (10)** marks.
2. Answer **all** questions.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

FCX048

1. (a) Using a non-programmable calculator evaluate:

(i) $\dfrac{\sqrt[3]{0.3854}(12.48)^2}{(0.04382)^4(\sqrt[5]{631.7})}$, correct to seven significant figures.

(ii) $\sum_{x=1}^{6} e^x \ln(1+x)^{\frac{2}{3}}$, giving your answer to three significant figures.

(b) Cosmic – ray bombardment of the atmosphere produces neutrons, which in turn react with nitrogen to produce radioactive Carbon – 14 $\left({}^{14}C\right)$. Radioactive ${}^{14}C$ enters all living tissues through Carbon dioxide, which is first absorbed by plants. As long as a plant or animal is alive, ${}^{14}C$ is maintained in the living organism at a constant level. Once the organism dies, however, ${}^{14}C$ decays according to the equation $A = A_0 e^{-0.000124}$ where A is the amount present after t years and A_0 is the amount present at time t = 0. If 500 milligrams of ${}^{14}C$ are present at the start, use a calculator to:

(i) Find how many milligrams will be present in 15,000 years correct to two decimal places.

(ii) Calculate the number of years it takes for 1.89 milligrams to remain.

2. (a) If $2\cosh y - 7\sinh x = 3$ and $\cosh y - 3\sinh^2 x = 2$, find the real values of x and y in logarithmic form that satisfy the two equations.

(b) (i) Verify that $\cosh 5x + \cosh 3x - 2\cosh x = 16\sinh^2 x \cosh^3 x$.

(ii) Show that $\tanh\left(\frac{1}{2}\ln x\right)\tanh\left(\frac{1}{2}\ln x\right) = \dfrac{x^2 - 2x + 1}{x^2 + 2x + 1}$.

(iii) Find the value of x which satisfy the equation $h \circ g \circ f(x) = 0$ when $f(x) = x^2 - 3$, $g(x) = \log_e x$ and $h(x) = \sinh x$.

3. (a) Mary and Jane earn 15,000/= and 20,000/= a day respectively from tailoring. Mary can stitch 6 shirts and 4 trousers while Jane can stitch 10 shirts and 4 trousers per day. How many days should each work to produce at least 60 shirts and 32 trousers at minimum cost?

(b) A wheat flour company has factories at A and B which supply warehouses at C and D. The weekly factory capacities are 160 and 140 units respectively and the warehouse requirements are 70 and 120 units respectively. The cost of transportation of one unit of wheat flour from A to C is shs. 160 and from A to D is shs. 240. Similarly, the cost of transportation from B to C is shs. 200 and from B to D is shs. 260. Without drawing the graph:

(i) Find the objective function to be minimized by the company so as to supply tons of wheat flour to each warehouse.

(ii) Find the inequalities associated to the transportation problem.

4. (a) The results of the weight of potatoes obtained from each of 100 roots are summarized as follows:

Weight of potatoes per Root (in Kg)	Frequency
0 – 3	9
3 – 6	22
6 – 9	28
9 – 12	21
12– 15	17
15-18	3

Find the mode, median and standard deviation for the weight of the potatoes.

(b) Using the information in 4(a) above find:
(i) the class interval where the 40^{th} percentile is located,
(ii) the interquartile range,
(iii) the actual mean using the coding method with assumed mean A = 7.5.

5. (a) (i) By using set properties, simplify the.set expression $(A-B)' \cap (A' \cup B)$.

(ii) If M $= \left\{x : \frac{x}{x-2} \geq 0,\ x \in \Re\right\}$ find all members of M.

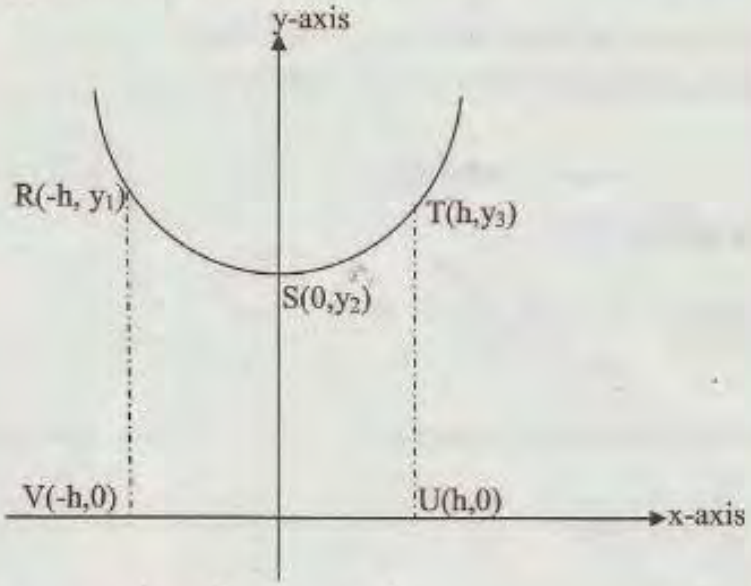
(b) A survey on the type of food crops grown in a certain village revealed that out of 210 families, 106 grow rice, 65 grow maize only, 48 grow rice and maize, 22 grow millet and maize and 14 grow rice and millet only. The number of families who grow rice only is twice the number growing millet only and 7 families interviewed grow non of these crops. Determine the number of families growing:
(i) all three crops.
(ii) exactly one crop.

6. (a) Given $f(x) = \sqrt{3x}$ and $g(x) = (x^2 - 1)^2$, find:
(i) the coefficient of x^2 from the product of $f \circ g$ and $g \circ f$,
(ii) the domain and range of $f \circ g$ and $g \circ f$.

(b) Determine the intercepts and asymptotes of the function $f(x) = \frac{x^2+4}{x^2-5x+6}$, and then draw the graph of $f(x)$.

7. (a) Use the Newton – Raphson method to approximate the root of $e^x(1+x) = 2$ correct to four decimal places by performing three iterations only. Apply $x_o = 0.1$.

(b) (i) Points $R(-h, y_1)$, $S(0, y_2)$ and $T(h, y_3)$ lie on the parabola $f(x) = kx^2 + bx + m$ which opens upwards. The lines $x = -h$, $x = h$, $y = 0$ and the parabola makes a region which is symmetrical to the line $x = 0$. Use the information given and the figure below to derive the Simpson's rule for approximation of area RSTUV.



(ii) Show that $\int_0^1 \frac{x^2}{(1+x^2)^2} dx = \frac{\pi}{8} - \frac{1}{4}$.

(iii) Use the results obtained in part (b) (ii) and the Simpson's rule with five ordinates to calculate the value of π correct to four decimal places.

8. (a) Determine the ratio which gives the point $\left(\frac{13}{2}, \frac{59}{8}\right)$ as an internal divider of a line segment joining points $(4, 3)$ and $(8, 10)$.

(b) (i) Find the equation of the tangent to the circle $x^2 + y^2 = 4$ at the point $(2\cos\theta, 2\sin\theta)$ in form $x\cos\theta + y\sin\theta = c$.

(ii) Find the equation of a circle passing through the points A(1, 3), B(2, 2), and C(5, 7).

(c) Lines l_1 and l_2 touch the circle $x^2 + y^2 - x + 5y + 10 = 0$ at points $P(-2,3)$ and $Q(5,-2)$ respectively. Find the angle between l_1 and l_2.

9. (a) Integrate $\int e^x \sin x dx$.

(b) Evaluate $\int_1^2 \frac{dx}{(x^2-2x+4)^{3/2}}$.

(c) Find the area of the region bounded by the functions $f(x)=x^3-8$, $f(x)=x^2$ and the x – axis giving your answer to six significant figures.

10. (a) If $u=g(x)$, use implicit differentiation to prove that $\frac{d}{dx}(u^n)=nu^{n-1}\frac{du}{dx}$ where n is a rational number.

(b) Given that $x=\frac{3(1+t^2)}{t-t^2}$ and $y=\frac{8t}{1-t^2}$, find $\frac{dy}{dx}$ in a simplified form.

(c) Use Taylor's theorem to expand $\sin(\frac{\pi}{3}+h)$ in ascending powers of h as far as the term in h^4.