THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2

ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Time: 3 Hours **Thursday, 14th February 2013 a.m.**

Instructions

1. This paper consists of sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

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SECTION A (60 marks)

Answer **all** questions in this section.

1. (a) Solve the equation $z^5 - 3i + 3 = 0$ expressing the roots in exponential form. Use the argument intervals $-\pi < \theta < \pi$ and integer $-2 \le n \le 2$ for the required roots.

(b) (i) If $z_1 = 2 + i$ and $z_2 = 3 - 2i$ evaluate $\left|\dfrac{2z_2 + z_1 - 5 - i}{2z_1 - z_2 + 3 - i}\right|$.

(ii) If z is a complex number, find the locus in polar form represented by the equation $|z - 1| = 3$ and hence determine the equation for the modulus.

(c) Use De Movie's theorem to prove that $\cos^4\theta = \frac{1}{8}\cos 4\theta + \frac{1}{2}\cos 2\theta + \frac{3}{8}$.

(d) If $\omega = 9 - 12i$, express $\omega^{0.5}$ as $a + ib$ where a and b are real numbers.

(15 marks)

2. (a) Given that P = "You like Economics",
Q = "You like Geography" and
R = "You like Advanced Mathematics".

Express the following symbolically:
(i) You like Economics or Geography but not Advanced Mathematics.
(ii) You like Economics and Geography or you do not like Economics and Advanced Mathematics.
(iii) It is not true that you like Economics but not Advanced Mathematics.

(b) If Tabita will complete the Advanced Certificate of Secondary Education Examination (ACSEE) and obtain good credits then she will apply for BA Statistics in higher learning institution. Tabita will either appy for BA or obtain good credits. However, Tabita has good credits. Therefore, she will apply for BA (Statistics). Formulate the hypotheses and determine whether the argument is valid.

(c) (i) Using the laws of algebra in logic, verify that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology.

(ii) Find a sentence having the following truth table:

P	Q	R	S(P,Q,R)
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	T

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(d) If $\underline{\vee}$ denotes the exclusive 'or', that is $P \underline{\vee} Q = (P \vee Q) \wedge \sim (P \wedge Q)$, complete filling the following table:

P	Q	$P \underline{\vee} Q$
T	T	
T	F	
F	T	
F	F	

(15 marks)

3. (a) (i) If $\underline{p}$ and $\underline{q}$ are two vectors and θ is an angle between them, determine the component of vector $\underline{p}$ in the direction of vector $\underline{q}$.

(ii) Find the projection of $\underline{a} = 3\underline{i} + 2\underline{j} - 6\underline{k}$ in the direction of $\underline{b} = -\underline{i} + 7\underline{j} + 2\underline{k}$ (leave your answer in surd form).

(b) The points A, B and C have position vectors $\underline{a} = 3\underline{i} - \underline{j} + 4\underline{k}$, $\underline{b} = \underline{j} - 4\underline{k}$ and $\underline{c} = 6\underline{i} + 4\underline{j} + 5\underline{k}$ respectively.

(i) Find the position vector of the point R on $\overrightarrow{BC}$ such that $\overrightarrow{AR}$ is perpendicular to $\overrightarrow{BC}$.

(ii) Using the result in part (b) (i) find the perpendicular distance of A from the line $\overrightarrow{BC}$.

(c) A plane travelling at 400 mph is flying with a bearing of 50°. The wind speed from south is moving at 40 mph. If no correction is made for the wind, find the ground speed of the plane, final bearing and the degree showing the push of the wind to the plane (write your answer to 2 decimal places).

(d) Given $\underline{a} = q\underline{i} + 3\underline{j} - 5\underline{k}$ where q is an integer such that $q < 0$ and the modulus of $\underline{a}$ is thirteen, verify that $\underline{a}$ is not parallel to $39\underline{i} + 7\underline{j} - 10\underline{k}$.

(15 marks)

4. (a) (i) Solve the quadratic inequality $x^2 + 2x - 8 \geq 0$.

(ii) If α and β are the roots of the quadratic equation $2x^2 + 3x - 4 = 0$, find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$ without calculating the values of α and β.

(b) Prove whether $n^3 + 6n^2 + 2n$ is divisible by 3 when $n \geq 1$.

(c) (i) Find the inverse of matrix $A = \begin{pmatrix} 3 & 6 & 9 \\ 1 & 7 & 10 \\ 4 & 8 & 3 \end{pmatrix}$ and then show that $AA^{-1} = I$.

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(ii) Use the inverse matrix obtained in 4 (c) (i) to solve the equations

$3x+6y+9z=6$

$x+7y+10z=5$.

$4x+8y+3z=17$

(iii) If $T=\begin{pmatrix} x & -\sin\alpha & \cos\alpha \\ \sin\alpha & -x & 1 \\ \cos\alpha & 1 & x \end{pmatrix}$ and $|T|=\frac{81}{64}x$, find x.

(15 marks)

SECTION B (40 marks)

Answer any **two (2)** questions from this section. Extra questions will not be marked.

5. (a) (i) Verify that $\tan\alpha+\cot\beta=\frac{\tan\beta+\cot\alpha}{\tan\beta\cot\alpha}$.

(ii) Prove that $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\text{cosec}\theta-\cot\theta$.

(b) Solve the equation $4\cos\theta-3\sec\theta=2\tan\theta$ for $-180°\le\theta\le180°$.

(c) Express $\cos(2\tan^{-1}x)$ as an algebraic expression in x free of trigonometric or inverse trigonometric functions.

(d) Show that the expansion of $\ln\sqrt{\frac{x+1}{x-1}}$ in ascending powers of x is $x+\frac{x^3}{3}+\frac{x^5}{5}.....$

(20 marks)

6. (a) The continuous random variable X has a probability density function $f(x)=\frac{3}{4}(1+x^2)$ where $0\le x\le1$. If $E(X)=\mu$ and $\text{var}(X)=\sigma^2$, find $P(|x-\mu|<\sigma)$.

(b) If the probability that an individual suffers a bad reaction from injection of a given serum is 0.001, determine the probability that out of 3000 individuals,

(i) exactly 4 individuals suffer a bad reaction.

(ii) more than 3 individuals suffer a bad reaction (write your answers to 3 decimal places).

(c) If the probability of a male birth is $\frac{1}{2}$, find the probability that in a family of 5 children there will be:

(i) at least 1 boy.

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(ii) at least 1 boy and at least 1 girl.

(d) What is the probability of picking 3 white, 4 black and 3 red shirts from a box containing 8 white, 5 black and 4 red shirts without replacement?

(20 marks)

7. (a) (i) Find the general solution of $y''-y'-6y=2(\sin 4x+\cos 4x)$.

(ii) Solve the equation $(x^2+y^3-4xy)dx+(3xy^2-2x^2+y^4)dy=0$.

(b) Find a suitable integrating factor and hence solve the differential equation $x\frac{dy}{dx}+3y=\frac{e^x}{x^2}$.

(c) (i) Radium has a half life of 1600 years. What percent of radium remains after 200 years?

(ii) Eliminate A and B completely from the exponential function $y=Ae^{3x}+Be^{-2x}$

(d) For a period of about 15 years, the rate of growth of a certain African country's gross domestic product (GDP) was predicted to vary between $+5\%$ and -1%. This variation was modeled by the formula $[5+4\cos\frac{1}{2}t]\%$, where t is the time in years. Find a formula for the GDP during the 15 years period.

(20 marks)

8. (a) Find the foci and the directrix of the ellipse $4x^2+16y^2=25$. Hence, identify the major axis of the ellipse where the foci are located.

(b) (i) Find the equation of the tangents to the curve $\frac{y}{x^2}=\frac{-1}{8}$ that will pass through the point $(1,1)$.

(ii) Show that a curve defined by the parametric equations $x=\frac{2}{3}t+7$ and $y=5t-1$ is a straight line.

(c) If $y^2=16-(x-1)^2$ is an equation of a circle, verify whether its radius is $\cos\theta\pm\sqrt{\cos^2\theta+15}$ where θ is an angle made by the radius and the polar axis.

(d) (i) Convert $r(1-2\sin\theta)=2$ into cartesian form.

(ii) Sketch the graph of $r=1+2\sin\theta$.

(20 marks)

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