THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1

(For Both School and Private Candidates)

Time: 3 Hours **Monday, 04th May 2015 a.m.**

Instructions

1. This paper consists of **ten (10)** questions, each carrying **ten (10)** marks.
2. Answer **all** questions.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).

$\overrightarrow{BA} = \overrightarrow{OA} + \lambda\overrightarrow{AB}$.

4. (a) Using a non-programmable calculator:

(i) Calculate $\log_e\left(e^4 + 2\ln 5\right) + \log 5$ and write your answer to six decimal places.

(ii) Obtain the value of $\sqrt{\dfrac{(4.03)^2 \times (814765)^{\frac{1}{10}}}{\sqrt{5}}}$ to three significant figures.

(iii) Find the value of $\left(\dfrac{{}^{6}C_3 \times \ln 2}{\sqrt[3]{43}}\right) \times \begin{vmatrix} 2e & \ln 2 \\ e & \ln 2 \end{vmatrix}$ to four decimal places.

(b) Evaluate $\sum_{r=1}^{4} e^{3r}\left(1 + (r+1)\ln r\right)$ to four significant figures.

2. (a) (i) Express $4\cosh\theta + 5\sinh\theta$ in the form $r\sinh(\theta + \alpha)$ giving the values of r and $\tanh\alpha$.

(ii) Prove that $\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$.

(b) (i) Show that $\frac{1}{2}\sinh(2\ln x)\cosh(2\ln x) = \frac{1}{8x^4}(x^8 - 1)$.

(ii) Find the possible values of $\sinh x$ if $\begin{vmatrix} \cosh x & -\sinh x \\ \sinh x & \cosh x \end{vmatrix} = 2$. (Leave your answer in surd form)

(c) Sketch the graph of $y = \sinh^{-1} x$ and state its domain and range.

3. (a) A company owns two mines. Mine A produces 1 ton of high grade ore, 3 tons of medium grade ore and 5 tons of low grade ore each day; and mine B produces 2 tons of each of the three grades of ore each day. The company needs 80 tons of high grade ore, 160 tons of medium grade ore and 200 tons of low grade ore. How many days should each mine be operated if it costs shs 200,000/= per day to operate each mine?

(b) A sugar company ships sugar from two origins S_1 and S_2 to three market centers M_1, M_2 and M_3. The table showing the available tons of sugar and the required tons together with the unit transportation cost in shillings is shown below:

	M_1	M_2	M_3	Available
S_1	20	10	5	220
S_2	10	25	30	100
Requirement	120	80	120	

(i) Use the given information in the table to formulate the objective cost function Z to be minimized.

(ii) Write down all equalities and inequalities of the transportation problem.

(iii) Verify whether the transportation problem in 3(b) is a balanced one or not. Use X_{ij}'s to denote the amount transported from source i to destination j.

4. Kamunonge cooperative farm with 20 branches each recorded one among the following sales of wheat last month: 6.1, 11.0, 22.3, 34.8, 37.5, 34.5, 29.4, 10.9, 17.3, 5.4, 3.2, 15.6, 22.6, 21.3, 20.5, 31.3, 47.9, 46.3, 41.4 and 48.2. Group the data into class intervals 0 – 10, 10 – 20, etc. and determine;

(a) (i) Mode of the data correct to 4 significant figures.
(ii) Median of the data.
(iii) The standard deviation correct to 4 significant figures.

(b) The lower and upper quartiles.

5. (a) (i) Use a Venn diagram to show that $(A \cap B) \cup (A' \cap B) = B$.

(ii) Find the members of set R where $R = \left\{ x : \frac{x^2 - 9}{x^2 - 1} \leq 0, \quad x \in \mathbb{R} \right\}$

(b) Use the basic properties of set operations to simplify the following:
(i) $(A \cap B) \cup (A - B)$.

(ii) $\left[(A \cup B) \cap (A \cap B) \right]'$

(c) In a bunch of twenty flowers, twelve are yellow and nine are red. If four of the flowers are neither yellow nor red, how many of the flowers are both yellow and red? (Use Venn diagram).

6. (a) If $f : x \to 5x + 4$ and $g : x \to 6x - k$.

(i) Determine the value of k for which $f \circ g(x) = g \circ f(x)$.

(ii) Prove that $f \circ (f \circ f(x)) = 125x + 124$.

(b) Draw the graph of $\frac{2x^2}{x^2 - 9}$.

7. (a) Starting with $x_0 = -1$, approximate the root of $f(x) = x + e^x$ in four iterations using the Newton-Raphson method. All your iterations should be presented in five significant figures.

(b) (i) Apply both Simpson's and Trapezium rule with eleven ordinates to find an approximate value of $\int_0^1 \sin(1+\sqrt{x})dx$. Give your answers correct to four decimal places.

(ii) Why does Simpson's rule said to be more efficient than trapezium?

8. (a) (i) Sketch the diagram of the locus of points which move such that they are equidistant from two intersecting lines.

(ii) Find the equations of bisectors to two intersecting lines whose equations are $6x-8y=-7$ and $4x+3y=12$.

(iii) Find the equation of the locus of points which is equidistant from the lines $y=2x$ and $2x+4y-3=0$.

(b) Determine the distance of the point $(8,-6)$ from the line $2x+5y+34=0$.

9. (a) Integrate $\int \sec^3 x\,dx$.

(b) Evaluate $\int_{-1}^{0}\left(\frac{2x+3}{x^2+2x+4}\right)$

(c) Find the area of the region bounded by the graphs of $y=\sin x$ and $y=\cos x$ between $x=0$ and $x=\frac{\pi}{2}$ (leave your answer in surd form).

10. (a) If $y=(1+2t)^3$ and $x=t^2$, find $\frac{dy}{dx}$.

(b) Find $\frac{d}{dx}\left(\tan\sqrt{6x^2+2}\right)$.

(c) (i) If $U=x^2e^{\frac{y}{x}}$, find dU.

(ii) Show that $(3x^2y-2y^2)dx+(x^2-4xy+6y^2)dy$ can be written as an exact differential equation of a function $\phi(x,y)$ and find this function.