THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1
(For Both School and Private Candidates)

Time: 3 Hours **Wednesday, 08th May 2019 a.m.**

Instructions

1. This paper consists of **ten (10)** questions each carrying **ten (10)** marks.
2. Answer **all** questions.
3. **All** necessary working and answers for each question done must be shown clearly.
4. Mathematical tables and non-programmable calculators may be used.
5. Cellular phones and any unauthorized materials are **not** allowed in the examination room.
6. Write your **Examination Number** on every page of your answer booklet(s).





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1. (a) By using a non-programmable calculator:
   (i) Calculate $\log_e\left(e^4 + 2\ln 5\right) + \log 5$ (Give your answer correct to six decimal places).
   (ii) Obtain the value of $\sqrt{\frac{(4.03)^3 \times (814765)^{0.5}}{\sqrt{5}}}$ correct to three significant figures.

   (b) The monthly salaries in Tanzanian shillings for 20 employees of KNCU are 260,000.00, 170,000.00, 85,000.00, 505,000.00, 129,000.00, 89,000.00, 220,000.00, 157,000.00, 103,000.00, 480,000.00, 790,000.00, 600,000.00, 340,000.00, 144,000.00, 128,000.00, 90,000.00, 102,000.00, 185,000.00, 219,000.00 and 195,000.00. Use the statistical functions of the scientific calculator to calculate:
   (i) the mean ($\bar{x}$) and
   (ii) the standard deviation ($x\delta n$).

2. (a) Solve the equation $\text{cosech}^{-1}(x) + \ln x - \ln 3 = 0$.
   (b) Given that $\sinh x = \tan\theta$, prove that $x = \ln(\sec\theta + \tan\theta)$.
   (c) Use the hyperbolic functions substitution to find $\int \frac{1}{\sqrt{(x^2 + 8x + 25)}}\,dx$.

3. Mr. Masumbuko has two traditional stores A and B for storing groundnuts. He stored 80 bags in A and 70 bags in B. Two customers C and D placed orders for 35 and 60 bags respectively. The transport costs per bag from each store are summarized in the following table:

From	To	
	C	D
A	8	12
B	10	13

   (a) How many bags of groundnuts should the farmer deliver to each customer in order to minimize the transportation cost?
   (b) Determine the minimum cost of transport.

4. (a) The sum of 20 numbers is 320 and the sum of the squares of these numbers is 5840.
   (i) Calculate the mean and standard deviation of 20 numbers.
   (ii) If one number is added to the 20 numbers so that the mean is unchanged, find this number and show whether the standard deviation will change or not.

   (b) A watchman at Mlimani city shopping centre recorded the length of time to the nearest minute that a sample of 131 cars was parked in their car park. The results were as follow:

Time (minutes)	5 – 10	11 – 16	17 – 22	23 – 28	29 – 34	35 - 40
Frequency	15	28	37	26	18	7

   (i) Calculate the median time correct to four significant figures.
   (ii) By using the coding method and the assumed mean $A = 19.5$, calculate the mean in two decimal places.

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5. (a) Use set properties to prove that for any non-empty sets A and B, $(A \cap B') \cup (B \cap A') = (A \cup B) - (A \cap B)'$.

(b) A student at the Sokoine University of Agriculture made a study about the types of livestock in a nearby village. The student came up with the following findings: 82 villagers kept cattle, 110 villagers kept goats, 73 villagers kept pigs; 59 villagers kept cattle and goats, 53 kept goats and pigs; 32 kept cattle and pigs; 20 villagers kept all three types of livestock. If the village has 200 occupants, by using Venn diagram, find the number of villagers who kept;

(i) only one type of livestock,
(ii) only two types of livestock,
(iii) none of the livestock.

6. (a) (i) Mention any two properties of $f(x) = b^x$.

(ii) Draw the graph of $f(x) = \left(\frac{1}{2}\right)^x$ for $-3 \le x \le 3$.

(b) Given that $y = \frac{x^2 - 2x - 3}{x^2 - 4}$

(i) Find the vertical and horizontal asymptotes.
(ii) Sketch the graph of y.

7. (a) The value $A = \int_a^b f(x)\,dx$ represents the area under the graph of $y = f(x)$ between $x = a$ and $x = b$. Derive the trapezium rule with 6 ordinates to find an approximation of $A = \int_a^b f(x)\,dx$.

(b) Using the trapezium rule obtained in (a) (ii), approximate $\int_1^7 \frac{x^3}{1+x^4}\,dx$, correct to three decimal places.

(c) Evaluate the actual integral of $\int_1^7 \frac{x^3}{1+x^4}\,dx$ and then calculate the relative error in the approximation obtained in (b). (Give your answers correct to three decimal places)

8. (a) The vertices of rectangle ABCD are given by points $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ and $D(x_4, y_4)$. Derive the formula to calculate the area of the rectangle.

(b) Use the formula obtained in part (a) to find the area of the rectangle whose vertices are the points $A(1, 1)$, $B(3, 5)$, $C(-2, 4)$ and $D(-1, -5)$.

(c) Show that the line $3x - 4y + 14 = 0$ is a tangent to a circle $x^2 + y^2 + 4x + 6y - 3 = 0$.

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9. (a) If $I_n = \int \sec^n x\,dx$, obtain a reduction formula for I_n in terms of I_{n-2} and use it to integrate $\int \sec^5 x\,dx$.

(b) Find the length of the arc given by $x = a(\cos\theta + \theta\sin\theta)$ and $y = a(\sin\theta - \theta\cos\theta)$ between $\theta = 0$ and $\theta = 2\pi$.

10. (a) If $y = \left(\frac{1-x^2}{1+x^2}\right)^n$, show that $(1-x^4)\frac{dy}{dx} + 4nxy = 0$.

(b) If the minimum value of $f(x) = 2x^3 + 3x^2 - 12x + k$ is one-tenth of its maximum value, find the value of k.

(c) (i) If $f(x, y) = x^3y + e^{-xy^2}$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

(ii) If $z = x^2\tan^{-1}\left(\frac{y}{x}\right)$, find $\frac{\partial^2 z}{\partial x \partial y}$ at $(1,1)$.

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