THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/2

ADVANCED MATHEMATICS 2
(For Both School and Private Candidates)

Time: 3 Hours **Year: 2020**

Instructions

1. This paper consists of sections A and B with a total of **eight (8)** questions.
2. Answer **all** questions in section A and **two (2)** questions from section B.
3. Section A carries **sixty (60)** and section B carries **forty (40)** marks.
4. All work done in answering each question must be shown clearly.
5. NECTA's mathematical tables and non-programmable calculators may be used.
6. Cellular phones and any unauthorized materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer booklet(s).



SECTION A (60 Marks)

Answer **all** questions in this section.

1. (a) Eggs are packed in boxes of 500. On the average 0.7% of the eggs are found to be broken. Find the probability that in a box of 500 eggs;
   (i) exactly 3 eggs are broken.
   (ii) at least two eggs are broken.
   (Write your answers in four significant figures)

   (b) As an experiment, a temporary roundabout is constructed at the crossroads. The time, X in minutes, which vehicles have to wait before entering the roundabout is a random variable having the following probability density function:

$$f(x)=\begin{cases}0.8-0.32x, 0\le x\le 2.5\\ 0, \text{otherwise}\end{cases}$$

   Find the mean waiting time for vehicles and standard deviation for the distribution.

   (c) The mean weight of 600 male villagers in a certain village is 79.7 kg and the standard deviation is 6 kg. Assuming that the weights are normally distributed, find how many villagers weigh more than 90 kg.

   (d) How many possible combinations of six questions are there in an examination paper consisting of a total of eight questions?

2. (a) Use the laws of propositions of algebra to simplify $(p\wedge q)\vee[\sim r\wedge(q\wedge p)]$.

   (b) Describe the following argument in symbolic form and test its validity by using a truth table:
   "If he begs pardon then he will remain in school. Either he is punished or he does not remain in school. He will not be punished. Therefore, he did not beg pardon.

   (c) Construct an electrical network for the proposition $(p\wedge q)\wedge[(r\vee s)\wedge w]$.

3. (a) Find the unit vector perpendicular to both vectors $\underline{a}+\underline{b}$ and $\underline{a}-\underline{b}$ where $\underline{a}=3\underline{i}+2\underline{j}+2\underline{k}$ and $\underline{b}=\underline{i}+2\underline{j}-2\underline{k}$.

   (b) The area of a parallelogram is $5\sqrt{6}$ units. If the adjacent sides of the parallelogram are $\underline{i}-2\underline{j}+\lambda\underline{k}$ and $2\underline{i}+\underline{j}-4\underline{k}$ respectively, find the positive value of λ.

   (c) A particle is moving so that at any instant its velocity $\underline{v}$ is given by $\underline{v}=3t\underline{i}-4\underline{j}+t^2\underline{k}$. If the particle is at point $P(1,0,1)$ when $t=0$, find;
   (i) the displacement vector when $t=2$.
   (ii) the magnitude of the acceleration when $t=2$.

4. (a) If $z=a+ib$, prove that $z.\overline{z}$ is a real number for all complex number z.

   (b) Given that $z=\cos\theta+i\sin\theta$, express $\cos^4\theta$ as the sum of cosines of multiple of θ.

   (c) If $z=\cos\alpha+i\sin\alpha$, show that $\dfrac{1}{1+z}=\dfrac{1}{2}\left(1-i\tan\left(\dfrac{\alpha}{2}\right)\right)$.

SECTION B (40 Marks)

Answer **two (2)** questions from this section.

5. (a) (i) Simplify the expression $\frac{1}{\sqrt{x^2-a^2}}$ where $x = a\,\text{cosec}\,\theta$.

(ii) Prove that $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta} = 2\,\text{cosec}\,\theta$.

(b) (i) Express $2\cos\theta+5\sin\theta$ in the form of $R\sin(\theta-\alpha)$.

(ii) If $\cos\alpha-\cos\beta=m$ and $\sin\alpha-\sin\beta=n$, express $\cos(\alpha-\beta)$ in terms of m and n.

(c) Use t - substitution to find the general solution of the equation $3\cos\theta-4\sin\theta+1=0$.

6. (a) Use the principles of mathematical induction to prove that $3^{2(n+1)}-8n-9$ is divisible by 8.

(b) Find the inverse of the matrix $A=\begin{pmatrix}3 & -1 & 2\\ 2 & 3 & 1\\ 1 & 2 & -1\end{pmatrix}$.

By using the inverse matrix obtained in (b), find the values of x, y and z in the simultaneous equations $\begin{cases}3x-y+2z=11\\ 2x+3y+z=-1\\ x+2y-z=-6\end{cases}$.

7. (a) Solve the differential equation $y\frac{d^2y}{dx^2}+25=\left(\frac{dy}{dx}\right)^2$ given that $\frac{dy}{dx}=4$ when $y=1$, and $y=\frac{5}{3}$ when $x=0$.

(b) Solve the differential equation $xy^2+x^2y\frac{dy}{dx}=\sec^2 2x$.

(c) The rate at which atoms in a mass of a radioactive material are disintegrating is proportional to the number of atoms (N) present at any time t. If N_0 is the number of atoms present at time $t=0$, solve the differential equation that represents this information.

(d) If half of the original mass disintegrates in 152 days, find the constant of proportionality for the solution obtained in (c). (Give your answer to three significant figures).

8. (a) Find the equation of a tangent to the ellipse $4x^2 + y^2 = 6$ at $\left(\frac{1}{2}, \sqrt{5}\right)$ in the form $ax + by + c = 0$.

(b) The points $P\left(at_1^2, 2at_1\right)$ and $Q\left(at_2^2, 2at_2\right)$ lie on the parabola $y^2 = 4ax$. The tangents at the points P and Q intersect at R. Find the coordinates of R.

(c) Convert the following polar equations into Cartesian equations:

(i) $r^2 = 4\sin 2\theta$.

(ii) $r = 3(1 + \cos\theta)$.

(d) A curve is defined by the parametric equations $x = t^2$ and $y = \frac{2}{t}$ where $t \neq 0$. Show that the equation of the normal at the point $Q\left(p^2, \frac{2}{p^2}\right)$ is $p^4x - py + 2 = p^6$.