# THE UNITED REPUBLIC OF TANZANIA
# NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
# ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

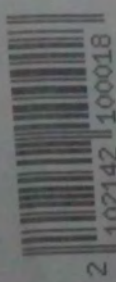
**142/1** **ADVANCED MATHEMATICS 1**

(For Both School and Private Candidates)

**Time: 3 Hours** **Year : 2021**

**Instructions**

1. This paper consists of **ten (10)** questions.
2. Answer **all** questions.
3. Each question carries **ten (10)** marks.
4. **All** necessary working and answers of each question done must be shown clearly.
5. NECTA's mathematical tables and non-programmable calculators may be used.
6. Cellular phones and any unauthorised materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer booklet(s).






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Answer **all** questions.

1. (a) Use a non-programmable scientific calculator to compute:

(i) the value of $\sqrt[5]{\ln(\log x)^4 + e^{5x}\sin 3x}$ correct to 7 decimal places, for $x = 0.2$ radian.

(ii) the derivative of $(x^2 - x + 4)^6$ at $x = 5$.

(iii) the modulus of $\dfrac{(4+3i)\times(3+4i)}{(3+i)}$ in radian. (Give your answers to four significant figures).

(b) The weight $x$ of 10 insects in mg are 1.20, 0.04, 1.40, 0.04, 0.716, 0.17, 1.20, 1.20, 2.40 and 3.00 respectively. Use a non-programmable calculator to compute $\sum x^2$ and $\delta x$ correct to three decimal places.

2. (a) Express $4\cosh\theta + 5\sinh\theta$ in the form $R\sinh(\theta + \alpha)$ and hence find the values of $R$ and $\tanh\alpha$.

(b) Show that $\cosh^{-1} x = \ln\left\{x + \sqrt{x^2 - 1}\right\}$.

(c) If $y = \dfrac{\cosh 2x}{1 + \sinh 2x}$ then find $\dfrac{dy}{dx}$.

3. (a) A farmer needs 10 kg of SA and 15 kg of CAN. He can buy bags containing 2 kg of SA and 1 kg of CAN or he can buy tins containing 1 kg of SA and 3 kg of CAN. If the cost of each bag and tin are 20/= and 50/= respectively, write down four inequalities representing the problem.

(b) Mr. Chapakazi has two storage depots. He stores 200 tons of rice at depot 1 and 300 tons at depot 2. The rice has to be sent to three marketing centres A, B and C. The demands at A, B and C are 150, 150 and 200 tons respectively. The transportation cost per ton from the deposits to each market center is shown in the following table:

| Deposits | Market Centre | | |
|---|---|---|---|
| | A | B | C |
| Depot 1 | 50 | 100 | 70 |
| Depot 2 | 80 | 150 | 40 |

How many tons of rice should be sent from the depots to each marketing centre so that the transportation cost is minimum?



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4. (a) If the standard deviation of the numbers $x_1, x_2, x_3, ....., x_n$ is 10, find the standard deviation of $2x_1+1, 2x_2+1, ....., 2x_n+1$.

(b) A classroom teacher measures the lengths of 50 students to the nearest centimeter. The results are summarized in the following table:

| **Length (cm)** | 31 – 35 | 36 – 40 | 41 – 45 | 46 – 50 | 51 - 55 | 56 – 60 |
|---|---|---|---|---|---|---|
| **Frequency (f)** | 3 | 6 | 17 | 10 | 9 | 5 |

(i) Calculate the first and third quartiles correct to two decimal places,

(ii) Calculate the $70^{th}$ percentile correct to one decimal place.

5. (a) Use the appropriate laws to simplify the expression $(A\cap B')\cup(A'\cap B)\cup(A\cap B)$.

(b) An investigation of eating habits of 110 rabbits revealed that 50 rabbits eat rice, 43 eat maize, 45 eat banana, 12 eat rice and maize, 13 eat maize and banana, 15 eat banana and rice and 5 eat all three types of food. Summarize the given information on a Venn diagram.

(c) Use the Venn diagram obtained in part (b) to find the number of rabbits which eat;

(i) only one type of food,

(ii) banana and maize but not rice,

(iii) none of the food.

6. (a) If $f(x)=\sqrt{x^2-9}$ and $g(x)=x-2$, find $f(g(x))$.

(b) Draw the graph of $f(g(x))$ obtained in part (a).

(c) Given that $y=\dfrac{x^2-9}{x-1}$;

(i) find the asymptotes of y,

(ii) draw the graph of $y$.

7. (a) Use the trapezium rule with 5 ordinates to find an approximate value for $\int_0^1 \dfrac{2}{1+x^2}dx$ correctly to 4 decimal places.

(b) Use Simpson's rule with 5 ordinates to find an approximation for $\int_0^1 \dfrac{2}{1+x^2}dx$ correct to four decimal places.

(c) Find the value of the integral $\int_0^1 \dfrac{2}{1+x^2}dx$ correct to four decimal places.

(d) Compare the actual value in part (c) with the approximated values obtained in part (a) and (b).

8. (a) (i) The point R($x$, $y$) divides a line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m:n$ internally. Derive the formula for the ratio theorem.

(ii) Use the ratio formula in (i) to find the coordinates of the point which divides the line joining the points $(5, -4)$ and $(-3, 2)$ internally in the ratio 1:2.

(b) (i) If the triangle has three vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, derive the formula to find its area.

(ii) Use the formula obtained in (b) (i) to find the area of the triangle with the vertices $A(3,1)$, $B(2k, 3k)$ and $C(k, 2k)$. Hence show that the vertices are collinear when $k = -2$.

9. (a) Find $\int x^3 \cos x \, dx$.

(b) Prove that $\int_0^1 x\sqrt{\frac{1-x^2}{1+x^2}}\, dx = \frac{\pi}{4} - \frac{1}{2}$.

10. (a) Show that there is a solution to the equation $x^3 - 6x^2 + 9x + 1 = 0$ between $x = -1$ and $x = 0$. Without using the table of values sketch the curve given by $y = x^3 - 6x^2 + 9x + 1$.

(b) Use Taylor's theorem to expand $(x+h)^{\frac{1}{2}}$ in ascending powers of $h$ up to the term containing $h^3$. Hence obtain the value of $\sqrt{10}$ giving your answer correct to five decimal places.



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