THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL OF TANZANIA
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

142/1

ADVANCED MATHEMATICS 1

(For Both Private and School Candidates)

Time: 3 Hours **Year: 2024**

Instructions

1. This paper consists of **ten (10)** questions.
2. Answer **all** questions.
3. **All** work done and answers of each question must be shown clearly.
4. NECTA'S Mathematical tables and non-programmable calculators may be used.
5. All writing must be in **black** or **blue** ink, **except** drawings which must be in pencil.
6. Communication devices and any unauthorised materials are **not** allowed in the examination room.
7. Write your **Examination Number** on every page of your answer booklet(s).

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1. (a) By using a non-programmable scientific calculator, compute the value of the following definite integrals correct to four decimal places:

(i) $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{1+\sin^2\theta} d\theta$

(ii) $\int_2^3 \frac{2x-3}{\sqrt{4x-x^2}} dx$

(b) Use a non-programmable scientific calculator, to calculate the mean and standard deviation of π, $\sqrt{2}$, e, $\sqrt{3}$, 1.414213, 2.718282, 3.1415, 1.732051 correct to six decimal places.

2. (a) By using the integrations by parts technique, evaluate the integral $\int_0^1 x\sinh 3x dx$, correct to three decimal places.

(b) If $\cosh x = 1+4\sinh x$, find the value of x.

(c) Prove that $\cosh^2 x+\sinh^2 x = \frac{1}{\operatorname{sech} 2x}$.

3. Mr. Mashauri needs 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively per jar and each jar is sold at Tsh.3,000. A dry product contains 1, 2 and 4 units of A, B and C respectively per carton and each carton is sold at Tsh.2,000. How much of each chemical should be purchased in order to minimize the cost?

4. (a) The mean of 200 numbers was 50. On rechecking it was found that 92 and 8 were incorrect. Find the correct mean if the wrong numbers were replaced by correct numbers 192 and 88.

(b) Use the coding method and assumed mean (A = 21) to calculate the mean and standard deviation for the following data:

Marks	0 - 6	6 - 12	12 - 18	18 - 24	24 - 30	30 - 36	36 - 42
Frequency	2	3	5	10	3	5	2

5. (a) By using the laws of algebra of sets, show that $(A\cup B)' \cap B = \Phi$.

(b) Use appropriate laws to simplify $[(A-B)-B]-(A-B)$.

(c) During the awarding day, 20 students received awards for academic excellence only, 30 students received awards for generosity only and 35 students received awards for smartness only. Also, 10 students received awards for generosity and academic excellent but not smartness, 60 received award for smartness and 55 students received award for generosity. If the number of students who received awards for smartness and academic excellence is equal to the number of students who received awards for generosity and smartness, use Venn diagram to find;

(i) the number of students who received awards for academic excellence.

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(ii) the total number of students.

6. (a) If $f(x) = x^3 - 12x - 7$, fill the following table with the missing values of $f(x)$.

x	-4	-3	-2	-1	0	1	2	3	4
$f(x)$									

Hence draw the graph of $f(x)$.

(b) If $f(x) = \dfrac{2x^3}{x^2 - 16}$,

(i) find the asymptotes.

(ii) sketch the graph of $f(x)$.

(iii) state the domain and range of $f(x)$.

7. (a) State two limitations of the Newton-Raphson formula.

(b) Use the Newton-Raphson formula to show that the kth root of a positive number A is given by $x_{n+1} = \dfrac{k-1}{k}\left[x_n + \left(\dfrac{A}{k-1}\right)x_n^{1-k}\right]$.

(c) Use the Trapezoidal rule with five ordinates to approximate the value of $\int_0^1 (1 + x^3)^{\frac{1}{2}}\, dx$ correct to three decimal places.

8. (a) What is the length of a tangent from (2, 2) to the circle $x^2 + y^2 + 6x - 2y = 0$.

(b) Find the equation of the normal to the circle $x^2 + y^2 - 24x + 14y + 63 = 0$ at the point $(9, 4)$.

(c) Find the distance of the point $(3, 2)$ from the normal line in part (b) correct to two decimal places.

9. (a) Find $\int \dfrac{5}{x^2 + x - 6}\, dx$.

(b) Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 x\, dx$ correctly to four decimal places.

(c) Find the length of the curve given by $x = 2\cos^3\theta$ and $y = 2\sin^3\theta$ between $\theta = 0$ and $\theta = \dfrac{\pi}{2}$.

10. (a) If $x^3y + y^3x = -2y$, find $\frac{dy}{dx}$ at $(-1,1)$.

(b) An object starts from rest and moves a distance of $g = \frac{1}{8}t^4 + \frac{1}{2}t^2$ *cm* in t seconds. By using this information find;

(i) the velocity of the object after two seconds.

(ii) the initial acceleration of the object.

(c) Differentiate $y = \tan^{-1}\left(\frac{a\sin x + b\cos x}{a\cos x - b\sin x}\right)$ with respect to x.