THE UNITED REPUBLIC OF TANZANIA
NATIONAL EXAMINATIONS COUNCIL
ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141

BASIC APPLIED MATHEMATICS
(For Both School and Private Candidates)

Time: 3 Hours

Tuesday, March 08, 2005 a.m.

Instructions

1. This paper consists of sections A and B.
2. Answer ***all*** questions in section A and ***four (4)*** questions from section B.
3. All work done in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae, slide rules and unprogrammable pocket calculators may be used.
5. Cellular phones are ***not*** allowed in the examination room.
6. Write your ***Examination Number*** on every page of your answer booklet(s).

This paper consists of 4 printed pages.

SECTION A (60 Marks)

Answer ALL questions in this section showing all necessary steps and answers.

1. (a) The equations of two straight lines L_1 and L_2 are $5x - 8y - 80 = 0$ and $8x + 5y - 128 = 0$ respectively. Show that L_1 and L_2 are perpendicular. **(02 marks)**

(b) The points A(p,q) and B(p – 1, 2q + 2) lie on the line $2x - y + 3 = 0$. Find the values of p and q. **(02 marks)**

(c) The points P(4, – 3), Q(– 3, 4), R(– 2, 7) and S are the vertices of a parallelogram. With the help of the coordinates of the midpoint of the diagonal PR, find the coordinates of point S. **(02 marks)**

2. The function f is defined as:

$$f(x) = \begin{cases} x \text{ when } 0 < x \leq 1 \\ x(x-2) \text{ when } 1 < x \leq 3 \end{cases}$$

Sketch the graph of f(x). **(06 marks)**

3. (a) The sum of the first and fifth terms of an arithmetic progression is 18 while the fifth term is 6 more than the third term. Find the sum of the first ten terms. **(03 marks)**

(b) A biscuit factory starts producing biscuits at the rate to 50,000 per hour. This rate of production decreases by 10 % every hour. Calculate the total number of biscuits produced in the first 3 hours. **(03 marks)**

4. (a) The cost of manufacturing a car is divided into the cost of materials, labour and other expenses in the ratio 5:2:1. Later on, the cost of materials increases by 30 %, that of labour by 40 % and other expenses decreases by 10 %. Determine the percentage increase in the cost of manufacturing the car. **(03 marks)**

(b) Y varies directly with the square root of X and inversely with the square root of Z. Given that $Y = 15$ when $X = 2\frac{1}{4}$ and $Z = \frac{16}{25}$, find the value of X when $Y = 56$ and $Z = \frac{2}{49}$. **(03 marks)**

5. (a) Solve the equation $\cos(40° + x) = \sin(2x - 10°)$ when $0 < x < 90°$. **(03 marks)**

(b) Show that $\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan\theta$ **(03 marks)**

6. (a) Solve the following equation:

$$4\log_a \sqrt{x} - \log_a 27x = \log_a x^{-2}$$ **(03 marks)**

(b) Find y in terms of x:

$$2\ln y - 3\ln x^2 = \ln \sqrt{x} + \ln x.$$ **(03 marks)**

7. (a) Find the value of $\int_0^{\frac{\pi}{4}} \cos^3 x \sin x \, dx$ by changing the limits. **(03 marks)**

(b) Evaluate $\int_0^1 \frac{4x \, dx}{(2-x^2)^{3/2}}$. **(03 marks)**

8. (a) Find the value of x which makes $\begin{bmatrix} x \\ 2 \\ 3 \end{bmatrix}$ perpendicular to $\begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}$. **(02 marks)**

(b) For any two non-zero vectors $\underline{a}$ and $\underline{b}$, if $\underline{a} - \underline{b}$ is perpendicular to $\underline{a} + \underline{b}$ show that $|\underline{a}| = |\underline{b}|$. **(04 marks)**

9. (a) Find the points on the curve $y = x^3 + 3x^2 - 6x - 10$ where the gradient is 3. **(03 marks)**

(b) Find $\frac{dy}{dx}$ for the following equation, $x^2 \sin y - y \cos x = 0$ **(03 marks)**

10. (a) Prove that $(\sin\theta + \operatorname{cosec}\theta)^2 = \sin^2\theta + \cot^2\theta + 3$. **(03 marks)**

(b) Solve the equation $4\cos\theta - 3\sec\theta = 2\tan\theta$ for $-180° \leq \theta \leq 180°$. **(03 marks)**

SECTION B (40 Marks)

Answer **four (4)** questions from this section showing all necessary steps and answers.

11. (a) A, B and C are the points (–1, 3, –1), (3, 5, –5) and (2, –2, 1). Find the

(i) Distance AB.

(ii) Cosine of the angle θ between AB and AC. **(04 marks)**

(b) If $\underline{a} = 4\underline{i} - 3\underline{j}$, $\underline{b} = 2\underline{i} + 4\underline{j}$ and $\underline{c} = 22\underline{i} - 11\underline{j}$, find the value of scalars m and n for which $m\underline{a} + n\underline{b} = \underline{c}$. **(04 marks)**

(c) Determine the value of λ so that $\underline{a} = 2\underline{i} + \lambda\underline{j} = \underline{k}$ and $\underline{b} = 4\underline{i} - 2\underline{j} - 2\underline{k}$ are perpendicular. **(02 marks)**

12. The terminal marks in the Basic Applied Mathematics examination obtained by 40 students in one of the secondary schools in Tanzania are as follows:

66,	87,	79,	74,	84,	72,	81,	78,	68,	74,
80,	71,	91,	62,	77,	86,	87,	72,	80,	77,
76,	83,	75,	71,	83,	67,	94,	64,	82,	78,
77,	67,	76,	82,	78,	88,	66,	79,	74,	64.

From the above data:

(a) Prepare a frequency distribution table with the lowest class interval of 60 - 64. (06 marks)

(b) Calculate the mean mark by using the coding method. (02 marks)

(c) Calculate the standard deviation correct to 2 decimal places. (02 marks)

13. A box contains 100 paper clips. 27 of the clips are too large and 16 of them are too small for the intended work. A paper clip is taken, judged and not replaced. A second clip is then treated similarly. Calculate the probability that:

(a) Both paper clips are acceptable for intended work. (04 marks)

(b) The first paper clip is too large and the second one is too small. (03 marks)

(c) One paper clip is too large and the other is too small. (03 marks)

14. (a) Solve the following system of equation by using the matrix method:

$$\begin{cases} 4x + 3y = 31 \\ 9y - x = 41 \end{cases}$$ (04 marks)

(b) If $A = \begin{pmatrix} 2 & 3 & -1 \\ -3 & 1 & 2 \\ 3 & -4 & -4 \end{pmatrix}$; find A^{-1}. (06 marks)

15. A person requires 10, 12 and 12 units of mineral elements A, B and C respectively for the diet. A liquid diet contains 5, 2 and 1 units of A, B and C respectively per can; and a dry diet contains 1, 2 and 4 units of A, B and C respectively per carton. If the liquid diet is sold at the price of sh. 3,000 per can and dry diet is sold at the price of sh. 2,000 per carton, how many cans and cartons should a person purchase to minimize the costs and meet the dietary requirements? (10 marks)

16. (a) Find the area under the curve $y = x^2 (x - 2)$ from $x = 0$ to $x = \frac{8}{3}$. (05 marks)

(b) A farmer encloses sheep in a rectangular field using hurdles for three sides and a long wall for the fourth side. If he has 100 m of hurdles, find the greatest area he can enclose. (05 marks)