# THE UNITED REPUBLIC OF TANZANIA
# NATIONAL EXAMINATIONS COUNCIL
# ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

**141**

## BASIC APPLIED MATHEMATICS
(For Both School and Private Candidates)

***Time: 3 Hours*** **2006 February, 07 Tuesday a.m.**

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*INSTRUCTIONS*

1. This paper consists of **sixteen (16)** questions in sections A and B.
2. Answer **all** questions in section A and **four (4)** questions from section B.
3. **All** work done in answering each question must be shown clearly.
4. Mathematical tables, mathematical formulae and non-programmable calculators may be used.
5. Cellular phones are **not** allowed in the examination room
6. Write your **Examination Number** on every page of your answer booklet(s).

## SECTION A (60 marks)

Answer **all** questions in this section showing all necessary steps and answers.

1. Boy scouts stationed at camp C, shown in figure 1 below, wish to cross a river to point B.

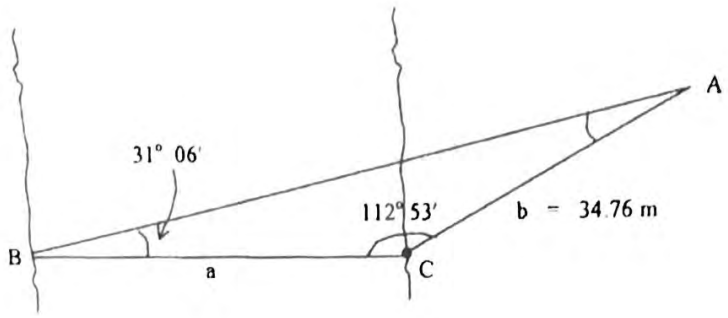



Fig. 1

They have found that angle ABC = 31°06′. Point A is a toilet stationed 34.76 m from camp C. Angle BCA = 112° 53′. How wide is the river between B and C?

**(6 marks)**

2. Juma wants to invest shs. 150,000 at a rate of 10 % compounded annually and accumulate the principal to shs. 250,000. Using a calculator with a log key, find how long this will take, given that:

$S = P(1 + i)^n$

Where i = interest
n = number of years
p = Principal

**(6 marks)**

3. (a) Find an equation (in the form $Ax + By + C = 0$) of the line which passes through the point (2, -1) and through the point of intersection of the line $3x - y + 7 = 0$ and $10x - 7y + 38 = 0$

**(3 marks)**

(b) Find the equation of the perpendicular bisector of the line joining the points A(2, -3) and B(6, 5).

**(3 marks)**

4. (a) Using the same xy plane, sketch the graphs of $f: x \rightarrow 5 - x$ and $g: x \rightarrow x$ and hence calculate the area of the triangle enclosed by the two graphs and the x axis. **(3 marks)**

(b) (i) Find $f^{-1}(x)$ if $f(x) = e^x$

(ii) Sketch the graph of $y = e^x$ and its inverse using the same xy plane.

(iii) What is your conclusion about the value of $\log_e N$ if $N \leq 0$? **(3 marks)**

5. (a) The sum of three consecutive numbers in an arithmetic progression (AP) is 33. The product of the same three numbers is 1232. Find the values of the three numbers. **($3^1/_2$ marks)**

(b) Given that the first term of a geometric progression (GP) is 2 and its common ratio is $\frac{1}{2}$, find the:

(i) $7^{th}$ term of the G.P. (ii) sum to infinity of this series. **($2^1/_2$ marks)**

6. (a) y is directly proportional to 2 and is inversely proportional to x. If $y = 3$ when $x = 2$ and $z = 1$;

(i) Deduce the equation connecting x, y and z, then evaluate y when $x = 10$ and $z = 5$.

(ii) Write y as a function of x when $z = 20$. **($3^1/_2$ marks)**

(b) The following table, (table 1) shows corresponding values of two variables F and C. Plot the values on a graph, hence estimate the relationship between the variables.

| C | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|
| F | 50 | 68 | 86 | 104 | 122 | 140 |

**($2^1/_2$ marks)**

Table 1

7. Given that $2A + B = 45°$, show that:

$$\tan B = \frac{1 - \tan^2 A - 2\tan A}{1 - \tan^2 A + 2\tan A}.$$

Hence find the value of $\tan(-15°)$ without using a calculator or mathematical tables. Simplify your answer, and rationalize the denominator. **(6 marks)**

8. (a) Solve for the real number x if

$$\log_{10} x = \log_5 2x.$$ **(3 marks)**

(b) Simplify $\dfrac{x}{y^{\frac{1}{2}} + x^{\frac{1}{2}}} + \dfrac{x}{y^{\frac{1}{2}} - x^{\frac{1}{2}}}$. **(3 marks)**

9. (a) Given that $f(x) = x^2 - \frac{1}{2}x + 3$, find the value of $f'(x)$ from first principles. **(3 marks)**

(b) Given that $y = \ln \dfrac{3x - 2}{x + 1}$, find $\dfrac{dy}{dx}$. **(3 marks)**

10. (a) A curve that passes through the origin has a gradient $2x - 1$. Find the equation of this curve in terms of x and y. **(3 marks)**

(b) $\int \sec^4 x \, dx$ by substituting $\tan x = t$, **(3 marks)**

## SECTION B (40 marks)

Answer **four (4)** questions from this section, showing all necessary steps and answers.

11. (a) Given the points A(2, - 1) and B(-3, 3), find:

(i) A vector from point A to point B in terms of unit vectors $\underset{\sim}{i}$ and $\underset{\sim}{j}$.

(ii) The length of vector $\overrightarrow{AB}$.

(iii) The unit vector in the direction of vector $\overrightarrow{BA}$ **(5 marks)**

(b) A line passes through the point (2, -1, 4) and is in the direction of vector $\underset{\sim}{i} + \underset{\sim}{j} - 2\underset{\sim}{k}$.

Find the:

(i) vector equation of the line.

(ii) angle the line makes with the positive x axis. **(5 marks)**

12. You are provided with the following frequency distribution table (table 2).

| i | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $x_i$ | 13 | 14 | 15 | 16 | 17 |
| $f_i$ | 1 | 4 | 12 | 2 | 1 |

Table 2

(a) Find the value of

(i) $\dfrac{\sum_{i=1}^{5} (X_i\ f_i)}{\sum_{i=1}^{5} f_i}$

(ii) $\sqrt{\dfrac{\sum_{i=1}^{5} (X_i - \overline{X})^2 f_i}{\sum_{i=1}^{5} f_i}}$ where

$$\overline{X} = \frac{\sum_{i=1}^{5} (x_i\ f_i)}{\sum_{i=1}^{5} f_i}$$

**(6 marks)**

(b) Find the: (i) median of the frequency distribution.
(ii) mode of the frequency distribution. **(4 marks)**

13. (a) Two dice are thrown together. What is the probability of a score of an eight? **(4 marks)**

(b) Two of my friends and I, play a game of pure chance three times. What is the probability of me winning:

(i) every time? (ii) only the third time? **(6 marks)**

14. (a) If T is a linear transformation such that:

$$T = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ and}$$

$$T(x, y) = (3y, 5x).$$

Find the matrix T hence evaluate T (0, 0). **(5 marks)**

(b) Use the inverse matrix method to solve:

$$2y + 3x - 15 = 0$$

$$2x - 20 + 3y = 0$$

**(5 marks)**

15. A certain farmer wants to use part of his shamba to plant cabbages and potatoes. He divides that part of his shamba into several equal sized portions. The farming of cabbages will cost shs. 24,000 per portion and potatoes shs. 8,000 per portion. The minimum funds which can be used in farming the two crops is shs. 240,000. Cabbages require 10 man hours per portion, while potatoes require 200 man hours per portion. The estimated profit is shs. 16,000 per portion of cabbages and shs. 12,000 per portion of potatoes.

(a) How should he allocate the expected shamba for maximum profit?

(b) What is the maximum profit? **(10 marks)**

16. (a) At time t, the position vectors of two particles P and Q are given by

$$\vec{P} = 2t\,\underset{\sim}{i} + (3t^2 - 4t)\underset{\sim}{j} - t^3\,\underset{\sim}{k}$$

$$\vec{Q} = t^3\,\underset{\sim}{i} - 2t\underset{\sim}{j} + (2t^2 - 1)\,\underset{\sim}{k}$$

Find the velocity and acceleration of Q relative to P when $t = 3$. **(5 marks)**

(b) A particle of unit mass moves so that its position vector $\underset{\sim}{r}$ at time t seconds is given by

$$\underset{\sim}{r} = (\cos t)\,\underset{\sim}{i} + (\sin t)\,\underset{\sim}{j} + \tfrac{1}{2}t^2\,\underset{\sim}{k}$$

Find the:

(i) momentum at time t.
(ii) kinetic energy at time t.
(iii) force acting on the particle at time t.
(iv) power exerted by the force in 16.(b)(iii) above at time t. **(5 marks)**