# THE UNITED REPUBLIC OF TANZANIA
# NATIONAL EXAMINATIONS COUNCIL
# ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141

## BASIC APPLIED MATHEMATICS
(For Both School and Private Candidates)

**Time: 3 Hours** **Monday, 05th May 2014 a.m.**

### Instructions

1. This paper consists of **ten (10) compulsory** questions. Each question carries **ten (10)** marks.
2. All work done in answering each question must be shown clearly.
3. Mathematical tables and non programmable calculators may be used.
4. Cellular phones are **not** allowed in the examination room.
5. Write your **Examination Number** on every page of your answer booklet(s).

1. (a) Solve the equation $\ln(2x+3)-3=\ln(x-5)$ and hence write your answer correct to 2 decimal places.

(b) Find the value of $t$ correct to 4 decimal places given that $3^{2t}=5^{t+1}$.

(c) Solve the quadratic equation $x^2+9x-2.718282=0$.

2. (a) Let $f(x)=\begin{cases} 2 & \text{for} \quad x<-2 \\ x^2+2x & \text{for} \quad -2\le x<1 \\ 4-x & \text{for} \quad x\ge 1 \end{cases}$

Find: (i) $f\left(\frac{1}{8}\right)$, (ii) $f(13)$ and (iii) $f(-32)$.

(b) A function is defined by $f(x)=\frac{2x-1}{x+1}$.

(i) Find its asymptotes and hence sketch the graph of this rational function.
(ii) State the domain and range of this function.

(c) Find the set of values of $(x, y)$ that satisfies the equations $x+y=3$ and $xy=2$.

3. (a) Given that the first term of a geometric progression is 2 and its common ratio is $\frac{1}{2}$;

(i) Write down the first four terms of the progression,
(ii) Find the $20^{th}$ term,
(iii) Find the sum to infinity of the series.

(b) The ages of a certain singers group form an arithmetic progression whose common difference is 4. If the youngest singer is 8 years old and the sum of the ages of all singers in the group is 168 years, find the number of singers in the group.

4. (a) Differentiate with respect to $x$ the following functions:

(i) $y=\frac{2e^{5x}}{3\sin x}$,

(ii) $y=x^2\sin 3x$,

(b) Find the slope of the curves:

(i) $f(x)=x^3-5x^2$ at the point $x=2$,

(ii) $x^2-3xy+2y^2-2x=4$ at the point (1, 3).

(c) The volume of air which is pumped into a rubber ball every second is $4\,\text{cm}^3$. Given that the volume of the ball is $v=\frac{4}{3}\pi r^3$ and that its radius ($r$) changes with the increase of air, find the rate of change of the radius when the radius is 6 cm.

5. (a) Evaluate the following integrals:

(i) $\int_2^5 (3x^2 - 5x)\,dx$,

(ii) $\int x^4\sqrt{x^5+3}\,dx$.

(b) Given that $\int_1^5 f(x)\,dx = 4$, evaluate $\int_1^5 (f(x)+3)\,dx$.

(c) Sketch the graph of the curve $f(x) = x(x-4)$ and hence find the area between the $x-$ axis and the curve.

6. During a biology practical, a random sample of 20 grasshoppers was selected and the length of each grasshopper recorded in centimeters as follows:
1.0, 1.0, 5.0, 4.0, 5.0, 5.0, 4.0, 2.0, 4.0, 2.0, 4.0, 2.0, 2.0, 3.0, 2.0, 3.0, 2.0, 3.0, 3.0, 3.0.

(a) Without grouping the data:
(i) Prepare a frequency table and a histogram for the length distribution.
(ii) Find the range, mode, median, mean and standard deviation.

(b) From part (a) (ii), indicate the measures of central tendency and the measures of dispersion.

7. (a) Evaluate:
(i) ${}^9P_4$ (ii) ${}^9C_4$

(b) In how many different ways can the letters in the word STATISTICS be arranged?

(c) In a pack of 52 playing cards, two cards which are not hearts are removed and not replaced. If the remaining cards are well shuffled, what is the probability that the next card drawn is a heart?

(d) Given that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{8}$ and $P(C) = \frac{1}{6}$, find:
(i) $P(A \cap B)$ if A and B are independent,
(ii) $P(A \cup C)$ if A and C are mutually exclusive events.

8. (a) Find the value of $\sin(A+B)$, given that $\sin A = \frac{3}{5}$, $\cos B = \frac{12}{13}$ and that A and B are both acute angles.

(b) Show that $\frac{\sec\theta + \operatorname{cosec}\theta}{1+\tan\theta} = \operatorname{cosec}\theta$.

(c) A triangular flower garden ABC has the angle $A\hat{B}C = 110°$. If AB is 50 m and BC is 40 m, find the length CA.

9. (a) Given $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -1 \\ -3 & 2 & 1 \\ 1 & 0 & -1 \end{pmatrix}$ and $C = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}$

Find: (i) AB (ii) BA (iii) CA and comment on the results.

(b) Solve the following system of equations by the inverse method.

$$\begin{cases} x + y + z = 3 \\ 5x - y - 3z = 1 \\ x - 3y + 2z = 0 \end{cases}$$

10. (a) What is a linear programming problem?

(b) Write 6 steps one would undertake in solving a linear programming problem graphically.

(c) Maximize the objective function $f(x, y) = 10x + 15y$ subject to the constraints:

$3x + 12y \leq 36$

$9x + 6y \leq 30$

$x \geq 0, y \geq 0$