THE UNITED REPUBLIC OF TANZANIA

NATIONAL EXAMINATIONS COUNCIL

ADVANCED CERTIFICATE OF SECONDARY EDUCATION EXAMINATION

141 BASIC APPLIED MATHEMATICS

(For Both School and Private Candidates)

Time: 3 Hours **ANSWERS** **Year: 2018**

Instructions

1. This paper consists of TEN questions.

2. Answer all questions.

1. (a) 1.51529
(b) Mean = 137.8481, standard deviation = 19.0593
(c) Determinant= 40
(d) t = 4.213, t = 0.787

2. (a) A step function f is defined on the set of real numbers such that
$f(x) = \{ 12x + 5$ if $x > 1$
$x - 4$ if $x \leq 1$
Find: $f(-1/2)$, $f(2)$ and $f(-3)$.

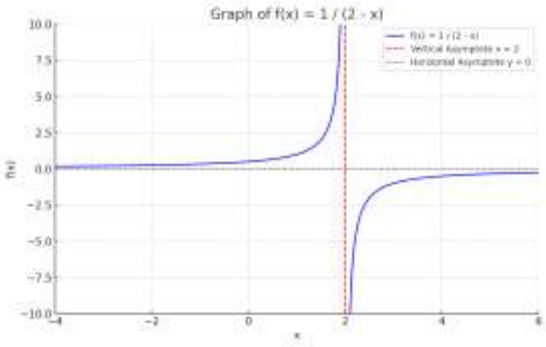
$f(-1/2)$: Since $-1/2 \leq 1$, use $f(x) = x - 4$
$f(-1/2) = (-1/2) - 4 = -4.5$

$f(2)$: Since $2 > 1$, use $f(x) = 12x + 5$
$f(2) = 12 \times 2 + 5 = 24 + 5 = 29$

$f(-3)$: Since $-3 \leq 1$, use $f(x) = x - 4$
$f(-3) = -3 - 4 = -7$

2. (b) Sketch the graph of $f(x) = 1 / (2 - x)$ and hence state its domain and range.
Rewriting: $f(x) = 1 / (2 - x) = -1 / (x - 2)$

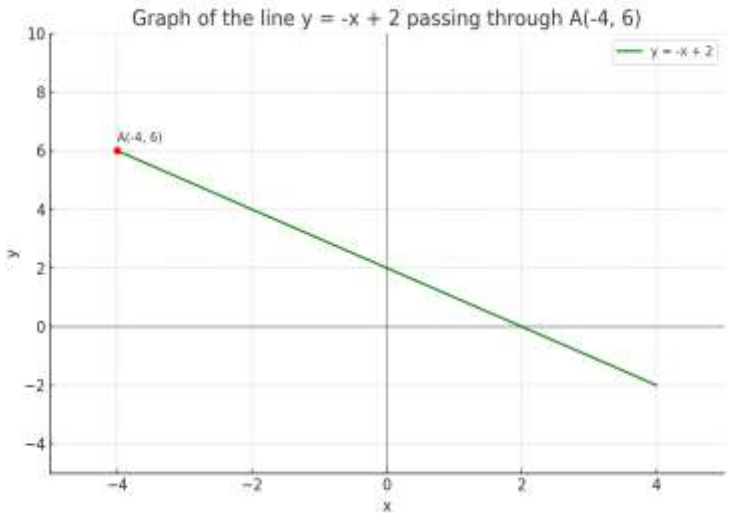
Vertical asymptote: $x = 2$
As x approaches 2 from the left, $f(x) \to -\infty$
As x approaches 2 from the right, $f(x) \to \infty$
Horizontal asymptote: $y = 0$
Domain: $x \in \mathbb{R}$, $x \neq 2$
Range: $y \in \mathbb{R}$, $y \neq 0$



2. (c) The line that passes through point A(−4, 6) has a slope of −1. Draw the graph of this line in the interval $-4 \leq x \leq 4$.

Using point-slope form:
$y - 6 = -1(x + 4)$

$y = -x - 4 + 6$
$y = -x + 2$
So the line is $y = -x + 2$ within the interval $x \in [-4, 4]$



3. (a) Solve the simultaneous equations
$x^2 - 2y = 7$
$x + y = 4$
by substitution method.

From second equation: $y = 4 - x$
Substitute into first:
$x^2 - 2(4 - x) = 7$
$x^2 - 8 + 2x = 7$
$x^2 + 2x - 15 = 0$
$(x + 5)(x - 3) = 0 \rightarrow x = -5$ or $x = 3$

If $x = -5 \rightarrow y = 4 - (-5) = 9$
If $x = 3 \rightarrow y = 4 - 3 = 1$

Solutions: $(-5, 9)$ and $(3, 1)$
(b) Calculate the sum of the series $\sum$ from $r = 3$ to 5 of $(-1)^{r+1} r^{-1}$

Evaluate each term:
$r = 3$: $(-1)^4(1/3) = 1 \times 1/3 = 1/3$
$r = 4$: $(-1)^5(1/4) = -1 \times 1/4 = -1/4$
$r = 5$: $(-1)^6(1/5) = 1 \times 1/5 = 1/5$

Sum $= 1/3 - 1/4 + 1/5$

Common denominator is 60:
$20/60 - 15/60 + 12/60 = 17/60$

Sum = 17/60

3. (c) The second and fifth terms of an A.P are $x - y$ and $x + y$ respectively, find the third term.
Let first term = a, common difference = d
Second term = $a + d = x - y$
Fifth term = $a + 4d = x + y$
Now subtract:
$(a + 4d) - (a + d) = x + y - (x - y)$
$3d = 2y \rightarrow d = 2y / 3$

From $a + d = x - y$:
$a = x - y - d = x - y - (2y / 3) = x - (5y / 3)$
Third term = $a + 2d = x - (5y / 3) + (4y / 3) = x - y / 3$
Third term is $x - y / 3$
4. (a) If $f(x) = x$, find dy/dx from first principles.

$f(x) = x \rightarrow f(x + h) = x + h$
$f(x + h) - f(x) = h$
$dy/dx = \lim h \rightarrow 0 (f(x + h) - f(x)) / h = h / h = 1$

$dy/dx = 1$

4. (b) Given the curve $f(x) = (x + 1)(x - 1)(2 - x)$

$f(x) = (x^2 - 1)(2 - x) = (-x^3 + 2x^2 + x - 2)$

(i) Find x and y intercepts

x-intercepts: Solve $f(x) = 0$
$(x + 1)(x - 1)(2 - x) = 0 \rightarrow x = -1, 1, 2$
y-intercept: $x = 0 \rightarrow f(0) = (0 + 1)(0 - 1)(2 - 0) = 1 \times (-1) \times 2 = -2$

(ii) Determine the maximum and minimum points of f(x)

$f(x) = -x^3 + 2x^2 + x - 2$
$f'(x) = -3x^2 + 4x + 1$
Set derivative to zero:
$-3x^2 + 4x + 1 = 0$
$3x^2 - 4x - 1 = 0$
$x = [4 \pm \sqrt{(16 + 12)}] / 6 = [4 \pm \sqrt{28}] / 6$

$= [4 \pm 2\sqrt{7}] / 6 = [2 \pm \sqrt{7}] / 3$

Second derivative: $f''(x) = -6x + 4$
Evaluate to classify nature

(iii) Sketch the graph

Plot points: x-intercepts at $x = -1, 1, 2$
y-intercept at $(0, -2)$
Shape of a cubic with negative leading coefficient: falls on right

5. (a) Integrate $\int 2x\sqrt{(x^2 + 3)}\, dx$

Let $u = x^2 + 3 \rightarrow du = 2x\, dx$
So integral becomes $\int \sqrt{u}\, du = (2/3)\, u^{3/2}$
Answer = $(2/3)(x^2 + 3)^{3/2} + C$

5. (b) Find the area of the region enclosed by the curve $y = x^2$ and the line $y = x$.

Find points of intersection: $x^2 = x \rightarrow x(x - 1) = 0 \rightarrow x = 0, 1$
Area = $\int_0^1 (x - x^2)\, dx = [x^2/2 - x^3/3]_0^1 = (1/2 - 1/3) = 1/6$

Area = 1/6 units^2

5. (c) Find the volume of revolution which is obtained when the area bounded by the line $y = 2x$, x-axis, $x = 1$ and $x = h$ is rotated about the x-axis.

Volume = $\pi \int_1^h (2x)^2\, dx = \pi \int_1^h 4x^2\, dx = 4\pi \int_1^h x^2\, dx$
$= 4\pi\, [x^3/3]_1^h = 4\pi(h^3 - 1)/3$
Volume = $(4\pi/3)(h^3 - 1)$

6. (a) Construct a frequency distribution table using equal class intervals of width 5 grams taking the lower class boundary of the first interval as 84.5.

Given data:
85, 87, 88, 89, 90, 91, 92, 92, 93, 94, 95, 95, 96, 96, 97, 98, 99, 100, 101, 101, 102, 103, 104, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123

Step 1: Determine number of intervals
Smallest value = 85
Largest value = 123
Range = 123 − 85 = 38
Class width = 5

Number of intervals = ceil(38 / 5) = 8

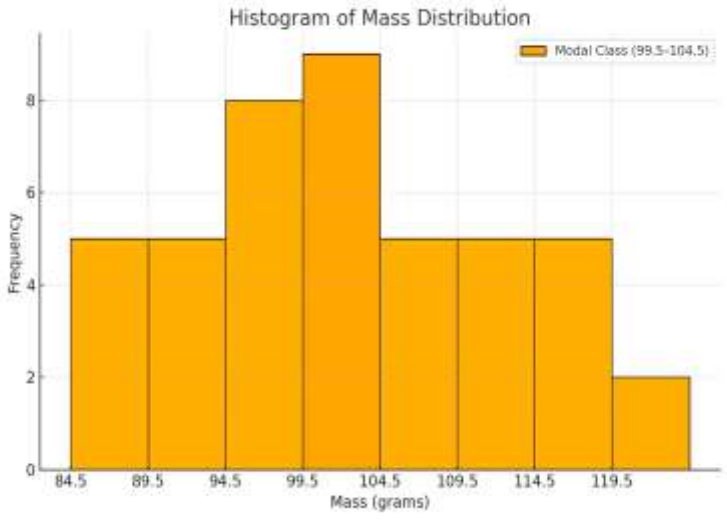
Class intervals and boundaries:
84.5 – 89.5
89.5 – 94.5
94.5 – 99.5
99.5 – 104.5
104.5 – 109.5
109.5 – 114.5
114.5 – 119.5
119.5 – 124.5

Now count the frequencies:

Class Interval	Tally	Frequency
84.5 – 89.5	||||	5
89.5 – 94.5	|||||	5
94.5 – 99.5	||||||||	8
99.5 – 104.5	|||||||||	9
104.5 – 109.5	|||||	5
109.5 – 114.5	|||||	5
114.5 – 119.5	|||||	5
119.5 – 124.5	||	2

Total = 44 values, confirmed.

6. (b) Draw the histogram to illustrate the data.



6. (c) Calculate the mode using the appropriate formula.

First, identify the modal class (class with highest frequency):
99.5 – 104.5 has highest frequency = 9

Let:
L = 99.5 (lower boundary of modal class)
$f_1 = 9$ (frequency of modal class)
$f_0 = 8$ (frequency before modal class = 94.5–99.5)
$f_2 = 5$ (frequency after modal class = 104.5–109.5)
h = 5 (class width)

Use formula:
$\text{Mode} = L + [(f_1 - f_0) / (2f_1 - f_0 - f_2)] \times h$
$\text{Mode} = 99.5 + [(9 - 8) / (2\times9 - 8 - 5)] \times 5$
$\text{Mode} = 99.5 + [1 / (18 - 13)] \times 5 = 99.5 + (1/5)\times5 = 99.5 + 1 = 100.5$

Mode = 100.5 grams

7. (a) Verify that ${}^8C_5 + {}^8C_2 = {}^9C_3$

Use values:
${}^8C_5 = 56$
${}^8C_2 = 28$
56 + 28 = 84
${}^9C_3 = 84 \rightarrow$ verified

7. (b) Two events A and B are such that P(A) = 1/3 and P(B) = 2/7

(i) Find $P(A \cup B)$ when A and B are mutually exclusive.
$P(A \cup B) = P(A) + P(B) = 1/3 + 2/7 = (7 + 6)/21 = 13/21$

(ii) Find $P(A \cap B)$ when A and B are independent.
$P(A \cap B) = P(A) \times P(B) = (1/3) \times (2/7) = 2/21$

7. (c) Two students are chosen at random from a class containing 20 girls and 15 boys.

(i) Find the probability that both are girls.
$P = (20/35) \times (19/34) = 380 / 1190 = 38 / 119$

(ii) One is a girl and the other is a boy.
Girl then boy: $(20/35)(15/34) = 300 / 1190$

Boy then girl: (15/35)(20/34) = 300 / 1190
Total = 600 / 1190 = 60 / 119

8. (a) (i) Without using a calculator, find the value of cos 15°

cos 15° = cos(45° − 30°)
= cos 45 cos 30 + sin 45 sin 30
= (1/√2)(√3/2) + (1/√2)(1/2)
= (√3 + 1)/(2√2)
= (√2)(√3 + 1)/4

(ii) Prove that sin(A + B)sin(A − B) = sin²A − sin²B

Use identity:
sin(A + B)sin(A − B) = ½[cos(B − B) − cos(2A)] = ½[1 − cos 2A]
But that's cosine form.
Alternatively use identity:
sin(A + B) = sin A cos B + cos A sin B
sin(A − B) = sin A cos B − cos A sin B
Product:
(sin A cos B + cos A sin B)(sin A cos B − cos A sin B)
= sin²A cos²B − cos²A sin²B
= sin²A − sin²B (if cos²B = 1 and cos²A = 1)

(iii) Sketch the graph of f(x) = sin x for $-2\pi \leq x \leq 2\pi$

Draw standard sine wave starting at (−2π, 0), going through 0 at 0, 2π, −2π
Max at π/2, −3π/2; Min at −π/2, 3π/2

8. (b) Solve the equation cos 2x + sin²x = 0, where $0° \leq x \leq 360°$

Use identity: cos 2x = 1 − 2sin²x
Then: 1 − 2sin²x + sin²x = 0 → 1 − sin²x = 0 → sin²x = 1
sin x = ±1

x = 90°, 270°

Solutions: x = 90°, 270°

9. (a) Three entrepreneurs R_1, R_2 and R_3 sell seedlings of two species A and B. If the sales in one month and prices paid (in Tsh) for each type are

$$S = \begin{matrix} A & B \\ 12 & 13 \\ 8 & 5 \\ 16 & 9 \end{matrix}$$

and

$$P = \begin{matrix} 2500 \\ 3500 \end{matrix}$$

respectively, find the total sales for each of the three entrepreneurs.

We are to compute the matrix multiplication $S \times P$:

$$S = \begin{matrix} 12 & 13 \\ 8 & 5 \\ 16 & 9 \end{matrix}$$

$$P = \begin{matrix} 2500 \\ 3500 \end{matrix}$$

Compute:

$R_1 = 12\times2500 + 13\times3500 = 30000 + 45500 = 75500$
$R_2 = 8\times2500 + 5\times3500 = 20000 + 17500 = 37500$
$R_3 = 16\times2500 + 9\times3500 = 40000 + 31500 = 71500$
Total sales:
R_1 = Tsh 75500
R_2 = Tsh 37500
R_3 = Tsh 71500

9. (b) Given matrix $A = \begin{matrix} 3 & -5 \\ 7 & -11 \end{matrix}$
Verify that $A^{-1}A = I$, where I is an identity matrix.

First, find A^{-1}.

Determinant of A:

$|A| = (3)(-11) - (-5)(7) = -33 + 35 = 2$

Adjoint of A =
$\begin{vmatrix} -11 & 5 \\ -7 & 3 \end{vmatrix}$

Now $A^{-1} = (1/|A|) \times \text{adj}(A) = (1/2) \times$
$\begin{vmatrix} -11 & 5 \\ -7 & 3 \end{vmatrix}$

Now multiply $A^{-1} \times A$:

$A^{-1} =$
$\begin{vmatrix} -11/2 & 5/2 \\ -7/2 & 3/2 \end{vmatrix}$

$A =$
$\begin{vmatrix} 3 & -5 \\ 7 & -11 \end{vmatrix}$

Compute matrix multiplication:

First row, first column:
$(-11/2 \times 3) + (5/2 \times 7) = -33/2 + 35/2 = 2/2 = 1$

First row, second column:
$(-11/2 \times -5) + (5/2 \times -11) = 55/2 - 55/2 = 0$

Second row, first column:
$(-7/2 \times 3) + (3/2 \times 7) = -21/2 + 21/2 = 0$

Second row, second column:
$(-7/2 \times -5) + (3/2 \times -11) = 35/2 - 33/2 = 2/2 = 1$

So $A^{-1} \times A =$
$\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = I$

Hence verified.

9. (c) Use Cramer's rule to solve:

$x + y + z = 6$

$2x + y - z = 1$
$x - y + z = 2$

Step 1: Coefficient matrix A:

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix}$$

Find determinant of A:

$|A| = 1(1\times1 - (-1\times-1)) - 1(2\times1 - (-1\times1)) + 1(2\times(-1) - 1\times1)$
$= 1(1 - 1) - 1(2 - (-1)) + 1(-2 - 1)$
$= 0 - 3 - 3 = -6$

Now construct matrices A_x, A_γ, $A\mathcal{z}$

A_x = replace first column with constants (6, 1, 2):

$$\begin{vmatrix} 6 & 1 & 1 \\ 1 & 1 & -1 \\ 2 & -1 & 1 \end{vmatrix}$$

$|A_x| = 6(1\times1 - (-1\times-1)) - 1(1\times1 - (-1\times2)) + 1(1\times(-1) - 1\times2)$
$= 6(1 - 1) - 1(1 + 2) + 1(-1 - 2) = 0 - 3 - 3 = -6$

A_γ = replace second column:

$$\begin{vmatrix} 1 & 6 & 1 \\ 2 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$|A_\gamma| = 1(1\times1 - (-1\times2)) - 6(2\times1 - (-1\times1)) + 1(2\times2 - 1\times1)$
$= 1(1 + 2) - 6(2 + 1) + 1(4 - 1) = 3 - 18 + 3 = -12$

$A\mathcal{z}$ = replace third column:

$$\begin{vmatrix} 1 & 1 & 6 \\ 2 & 1 & 1 \\ 1 & -1 & 2 \end{vmatrix}$$

$|A\mathcal{z}| = 1(1\times2 - (1\times-1)) - 1(2\times2 - 1\times1) + 6(2\times-1 - 1\times1)$
$= 1(2 + 1) - 1(4 - 1) + 6(-2 - 1)$

$= 3 - 3 - 18 = -18$

Now compute:
$x = A_x / |A| = -6 / -6 = 1$
$y = A_y / |A| = -12 / -6 = 2$
$z = A_z / |A| = -18 / -6 = 3$

Solution:
$x = 1$
$y = 2$
$z = 3$

10. (a) Mention any four applications of linear programming.

Production planning: Maximizing output or minimizing cost by allocating limited resources like labor, materials, and machines.

Transportation: Minimizing the cost of transporting goods from multiple sources to various destinations.

Diet formulation: Determining the cheapest combination of foods that satisfies all nutritional requirements.

Scheduling: Allocating time slots for machines, employees, or projects to minimize delay or idle time.

10. (b) Define the following terms in linear programming:

(i) Objective function
This is the function to be maximized or minimized in a linear programming problem. It usually represents profit, cost, or efficiency. Example: Maximize $Z = 3x + 4y$.

(ii) Constraints
These are the conditions or restrictions that the solution must satisfy. They are usually expressed as linear inequalities. Example: $x + y \leq 10$.

(iii) Feasible region
This is the set of all possible solutions that satisfy all constraints. It is usually a polygonal area on a graph, and any point within this region is a potential solution.

10. (c) A special take away fast lunch of food and drinks contains 2 units of vitamin B and 5 units of iron. Each glass of drinks has 4 units of vitamin B and 2 units of iron. A minimum of 8 units of vitamin B and 60 units of iron are served each day. If each serving of food costs 2000 Tsh and that of drinks costs 1600 Tsh; how much of the food and drinks are needed to be consumed in order to meet daily needs at a minimum cost?

Let
x = number of food servings
y = number of drink servings

Constraints:
$2x + 4y \geq 8$ (Vitamin B requirement)
$5x + 2y \geq 60$ (Iron requirement)
$x \geq 0, y \geq 0$

Objective function:
Minimize cost $C = 2000x + 1600y$

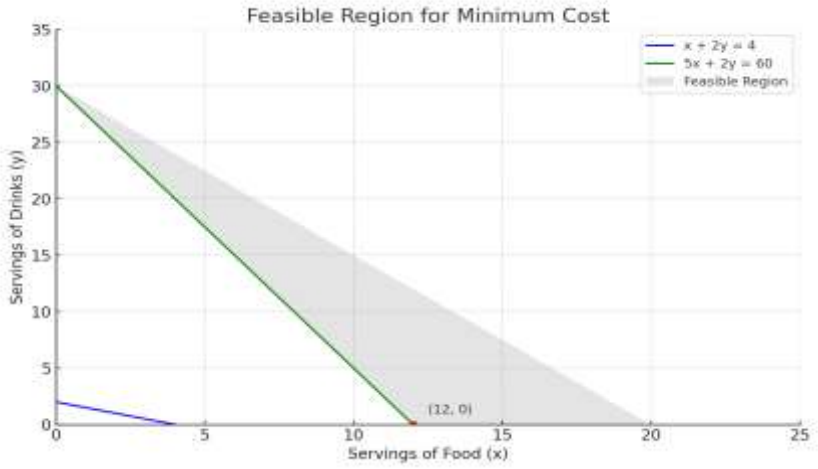
Now solve the system:

From $2x + 4y \geq 8 \rightarrow x + 2y \geq 4$
From $5x + 2y \geq 60$

Now solve $x + 2y = 4$ and $5x + 2y = 60$ simultaneously

Subtract the first from the second:

$5x + 2y - (x + 2y) = 60 - 4 \rightarrow 4x = 56 \rightarrow x = 14$
Substitute into first:
$14 + 2y = 4 \rightarrow 2y = -10 \rightarrow y = -5 \rightarrow$ invalid since $y \geq 0$



Try boundary points:

If $x = 0 \rightarrow$ from $x + 2y \geq 4 \rightarrow 2y \geq 4 \rightarrow y \geq 2$
from $5x + 2y \geq 60 \rightarrow 2y \geq 60 \rightarrow y \geq 30$

So y = 30 satisfies both when x = 0 → Cost = 2000×0 + 1600×30 = 48000

If y = 0 → from $x + 2y \geq 4 \rightarrow x \geq 4$
from $5x \geq 60 \rightarrow x \geq 12$
So x = 12 satisfies both → Cost = 2000×12 = 24000

Try x = 10 → check both:

$x + 2y \geq 4 \rightarrow 10 + 2y \geq 4 \rightarrow$ always satisfied
$5x + 2y \geq 60 \rightarrow 50 + 2y \geq 60 \rightarrow 2y \geq 10 \rightarrow y \geq 5$
Try y = 5
Check all:
$x + 2y = 10 + 10 = 20 \geq 4$ ✓
$5x + 2y = 50 + 10 = 60$ ✓
Cost = 2000×10 + 1600×5 = 20000 + 8000 = 28000

Try x = 12, y = 0 → Cost = 24000
This is the minimum

Minimum cost = Tsh 24000
Food servings = 12
Drink servings = 0